

Student Learning Extension Opportunities

Grade 9-Grade 12

Directions: These learning activities are provided for practice opportunities. Refreshing your memory of the concepts learned and keeping your mind engaged will help you maintain the skills you have learned. These learning activities are designed to provide practice over the course of the week, so spread out the work. We look forward to seeing you back in class soon.

WEEK ONE

Reading and Writing (Science and Social Studies Integration):

Week 1, Day 1

- Read a book at your reading level for thirty minutes. Keep track of your daily reading on the reading log below.
- Read the text, "Analyze This: Beauty products are big sources of urban air pollution."
- Answer the two questions from the text and complete the writing task.

Week 1, Day 2

- Read a book at your reading level for thirty minutes. Keep track of your daily reading on the reading log below.
- Read the text, "UCLA institute on kindness wants to promote a more humane world."
- Answer the two questions from the text and complete the writing task.

Week 1, Day 3

- Read a book at your reading level for thirty minutes. Keep track of your daily reading on the reading log below.
- Read the text, "The surprising differences between our view of the world and reality."
- Answer the two questions from the text and complete the writing task.

Week 1, Day 4

- Read a book at your reading level for thirty minutes. Keep track of your daily reading on the reading log below.
- Read the text, "Teens Health - Why Exercise Is Wise."
- Answer the two questions from the text and complete the writing task.

Week 1, Day 5

- Read a book at your reading level for thirty minutes. Keep track of your daily reading on the reading log below.
- Read the text, "Scientists At Work: Diving in underwater caves for new forms of life."
- Answer the two questions and complete the writing task.

Mathematics:

Week 1, Day 1

- For students enrolled in Algebra I or Geometry, complete your course-appropriate worksheet labeled *Algebra I* or *Geometry*.
- For students enrolled in Algebra II or higher, complete 10-12 problems on the *2019-2020 ACT Practice Test*.

Student Learning Extension Opportunities

Grade 9-Grade 12

Week 1, Day 2

- For students enrolled in Algebra I or Geometry, complete your course-appropriate worksheet labeled *Algebra I* or *Geometry*.
- For students enrolled in Algebra II or higher, complete 10-12 problems on the *2019-2020 ACT Practice Test*.

Week 1, Day 3

- For students enrolled in Algebra I or Geometry, complete your course-appropriate worksheet labeled *Algebra I* or *Geometry*.
- For students enrolled in Algebra II or higher, complete 10-12 problems on the *2019-2020 ACT Practice Test*.

Week 1, Day 4

- For students enrolled in Algebra I or Geometry, complete your course-appropriate worksheet labeled *Algebra I* or *Geometry*.
- For students enrolled in Algebra II or higher, complete 10-12 problems on the *2019-2020 ACT Practice Test*.

Week 1, Day 5

- For students enrolled in Algebra I or Geometry, complete your course-appropriate worksheet labeled *Algebra I* or *Geometry*.
- For students enrolled in Algebra II or higher, complete 10-12 problems on the *2019-2020 ACT Practice Test*.

Reading Log

Keep track of your daily reading.

Beginning Page	Ending Page	Title

ANALYZE THIS! // ENVIRONMENT

Analyze This: Beauty products are big sources of urban air pollution

Home cleaning and other products also turn out to be surprisingly large culprits



A large percentage of air pollution in cities such as Los Angeles, Calif., may be due to the use of common household products, one study finds. Each use of hairspray, glue and paint, for instance, has a small effect. But when used by millions of people, the effect on air quality can be large.

MATTGUSH/ISTOCKPHOTO

By **Lillian Steenblik Hwang**

June 27, 2018 at 5:30 am

People commonly use shampoo, hairspray, deodorant, cleaning sprays, paint and glues. Such products can dirty the air. In fact, they are as bad for urban air as is burning gasoline or diesel fuel. That's the finding of a recent study.

Each use of [these products](#) releases a mix of chemicals. Some of these belong to a class known as VOCs. That's short for volatile organic compounds (which means they are gases and contain

carbon). Many VOCs hang around in the outdoor air. Quite a few common VOCs come from products made from petroleum and other fossil fuels.

Certain VOCs can react with other chemicals in the air to create ozone. These VOCs can also help create particles known as “fine” (or small) particulates. Ozone is an ingredient of smog. That smog, along with high levels of fine particulates, can make it hard to breathe. This can be especially true for people who have lung problems, such as asthma. Ozone and particulates also can contribute to other lung problems, diabetes and heart disease.

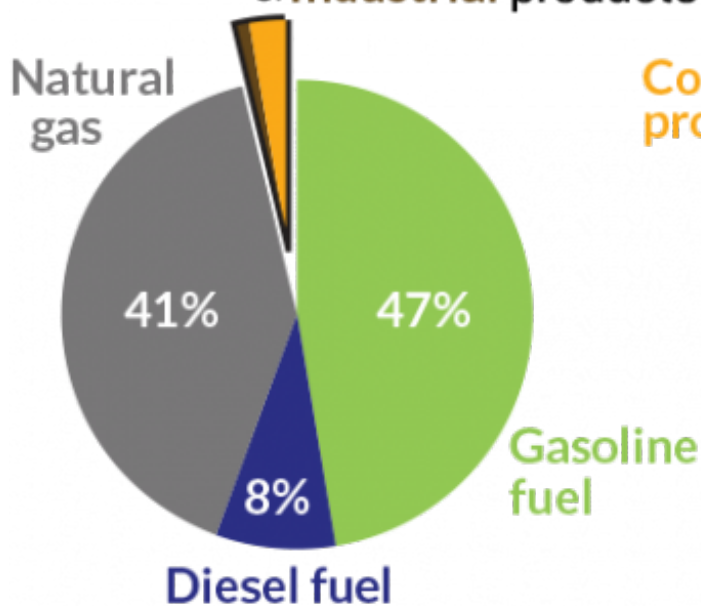
When people think of outdoor air pollution, their thoughts probably go to the exhaust gases spewed by cars and trucks. Or they might think of gases leaving factory smokestacks. But “sources of air pollution [in cities] are becoming more diverse,” Brian McDonald [told Science News](#). McDonald is a chemist. He works at the Cooperative Institute for Research in Environmental Sciences in Boulder, Colo. He shared his team’s new findings in February at a meeting of the American Association for the Advancement of Science in Austin, Texas.

On average, people use 15 times more gasoline and diesel, by weight, than VOC-emitting household products. But cosmetics, cleansers and other household products created 38 percent of the VOC emissions that this recent study found in city air.

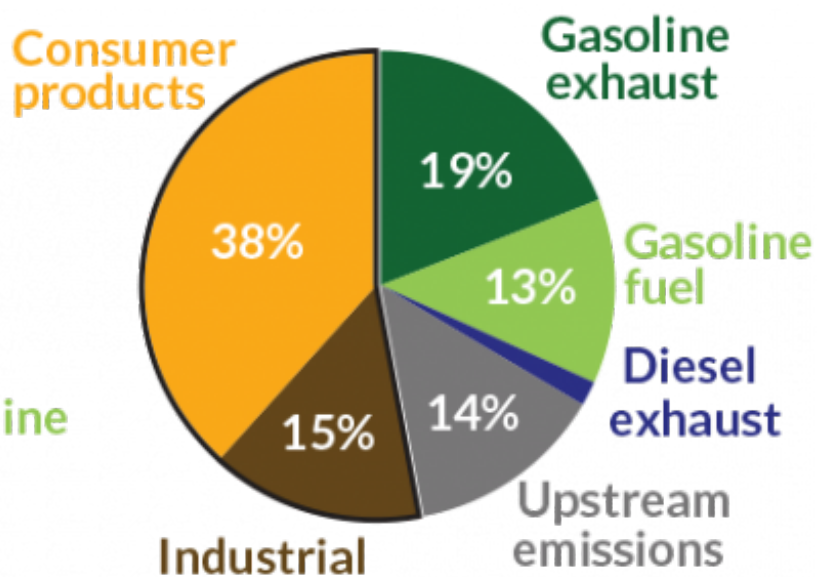
In fact, those products were bigger contributors than fossil fuels were to the creation of fine particulates, ozone and smog. Frequent use of these products by millions of people daily can really add up to a lot of air pollution.

Emission sources: Use versus contribution to smog

Consumer & Industrial products 4%



Product use (by mass)



Smog-forming emissions (by mass)

In Los Angeles, consumer goods such as bath products and cleaning chemicals make up a tiny amount of the sources releasing VOCs into the air. They have an enormous effect, however. For instance, they are responsible for as much VOC pollution in cities as is the burning of gasoline and diesel.

REPRINTED WITH PERMISSION FROM B.C. McDONALD ET AL., SCIENCE 359: 760 (2018)

CITATIONS

Citation:

Journal: B. McDonald et al. [Volatile chemical products emerging as largest petrochemical source of urban organic emissions](#). *Science*. Vol. 359, February 16, 2018, p. 760. doi:10.1126/science.aaq0524.

“Analyze This: Beauty products are big sources of urban air pollution”
Comprehension Questions

Answer the following questions, citing evidence from the text.

- 1) What percent of smog-forming emissions (by mass) comes from the use of consumer products?

- 2) Describe how you would represent the data in the pie charts as a bar graph, either with words or by drawing the graph.

Often scientific research is the first step in solving a problem. Yet, every solution has tradeoffs, both costs (not always monetary) and benefits for individuals and communities. Develop a proposal, based on evidence in the article, of what a household like yours could do to help solve the problem described in the article. In your proposal, describe the tradeoffs or costs to your family and the benefits of your action plan.

UCLA institute on kindness wants to promote a more humane world

By Los Angeles Times, adapted by Newsela staff on 10.09.19

Word Count **1,196**

Level **1070L**



Image 1. Four women smile as they stand on a mountain. Photo by: RawPixel

LOS ANGELES, California — A friendly smile. A food pantry donation. Or, a remarkable act of Los Angeles, California, benevolence — allowing a driver to cut in front of you.

Such acts of kindness have a self-serving upside, too, as science has conclusively shown they also make you healthier.

Now the University of California, Los Angeles (UCLA), is advancing that science with the September 25 launch of the world's first research institute on kindness. The institute blends multiple academic subject areas together. The institute will explore, for instance, how and why being nice to others reduces depression and the risk of cancer and cardiovascular disease.

Mental illness refers to a range of mental health conditions. They are disorders that can affect mood, thoughts and behavior. Examples include depression, anxiety and eating disorders. Many people have mental health concerns at some point. What distinguishes a mental illness, such as depression, from normal feelings such as sadness, is that mental illnesses cause ongoing stress. It

can also severely interfere with a person's ability to function. Most mental illnesses can be treated with medication and therapy.

Kindness Alters The Behavior Of Genes

Research by UCLA scientists has shown that mindfulness and kindness actually alter the behavior of genes. Mindfulness is the practice of focusing on the present aided by slow and deep breathing.

Mindfulness and kindness alter the behavior of genes by turning down those that promote swelling, which can lead to heart disease or certain cancers. At the same time, they turn up the activity of genes that protect against infections.

Training Tools

However, the ultimate goal of the UCLA Bedari Kindness Institute is to spread kindness and promote a more humane world. It will develop training tools to help practice kindness and spread them through online programs, public lectures, media outreach and a free app called UCLA Mindful. The app is already available. A \$20 million gift from the Bedari Foundation, established by philanthropists Jennifer and Matthew C. Harris, will provide money for the institute's research projects.



"In the midst of current world politics, violence and strife, the UCLA Bedari Kindness Institute seeks to be an antidote," said Darnell Hunt, the head of the UCLA branch of social sciences. The venture will be under this department.

Researchers agreed on an academic definition for kindness: an act that improves the welfare of others as an end in itself. When it comes to kindness, the intention — rather than the outcome — is key. In other words, it's the thought that counts, as the saying goes.

Kindness is complimenting someone to make them feel good, not to get what you want. It's sending a donation to a charity even if the check gets lost in the mail. It's thinking about the reason why a driver who cuts you off might be in a hurry.

Positive Effects On Well-Being

Daniel Fessler is a professor of anthropology at UCLA. He studies the development of human societies. He is also the institute's inaugural director. Fessler said that having kind thoughts increases how often you take kind actions. Both the thoughts and the experience of doing the actions have positive effects on the well-being of the person.

Already, a range of UCLA researchers are studying the types of questions that will be the basis of the institute's work. The work will focus on three themes: the roots of kindness, how to promote it and how to use it as a healing method to improve mental and physical health.

Fessler said humans have come to dominate the globe, despite their relatively small size. This is due to humans' unique ability to cooperate.

"As troubling as violence and cruelty are in our society, the actual level of positive cooperation is astounding at an evolutionary level," Fessler said. "Our species is a hyper-cooperative one. No other species is engaged in such a large level of cooperation among individuals who are not kin."

He noted, however, humans also have a long history of violent conflict and cruelty between groups. Aliza Luft is a sociologist at UCLA. She studies human society. Her research explores cultural factors that promote cruelty and lead some members of the dominant group to choose kindness instead. One example is the "righteous Gentiles." The righteous Gentiles were non-Jews who risked their lives to save Jews from Nazi oppression and the Holocaust during World War II.

Kindness Can Be Contagious

Fessler and other anthropologists are studying how kindness can be contagious, spreading among individuals and groups. In an experiment, people watched a video clip of someone showing kindness. They were more likely to donate money to a children's hospital than people who watched a video without kind actions. The researchers had given each participant \$5 and showed the videos. Then they turned their backs as each person decided whether to place any money in an envelope, which was padded to conceal the contents.

UCLA researchers also have shown that kindness can significantly ease depression and anxiety. Michelle Craske is a professor of the mind, mental illness and behavior. Craske shows that patients who received compassion training to encourage joy, gratitude, loving kindness and generosity, and also engaged in kind acts saw significant improvement in their mental health. The kind act could be as simple as offering to help co-workers on projects, for instance. The improvement in mental health lasted throughout the six months researchers followed the patients, she said.

Craske plans to start a similar research project with high school students who are at risk of depression in Imperial Valley, California. She also wants to expand efforts to help UCLA students. She hopes sharing the methods of mindfulness training will help combat what many experts say is a national rise in mental health problems among students. Craske is also developing virtual reality tools to replicate positive environments that can help boost people's sense of well-being.

Mindfulness Can Change The Brain

Michael Irwin is a psychiatrist, someone who diagnoses and treats mental illness. He also studies the nervous system and the immune system. Irwin and his co-workers have published several studies that found mindfulness and kindness actually change the brain and behavior of genes.

One study that is still in progress is looking at people who are caregivers to patients with Alzheimer's disease. Alzheimer's is a disease that affects the brain and causes extreme memory loss. The study found mindfulness training reduced problems with sleep and depression.

Arlene Winnick was a participant in the study. She said she initially doubted what she thought would be "woo woo" symbolic practices. However, she said the simple steps work, helping her sleep, relax and feel more energized as she deals with the stressful demands of caring for her partner with Alzheimer's.

To spread such healing is why Jennifer and Matthew Harris wanted to start the Bedari Kindness Institute. Matthew Harris thinks kindness is critically important to the preservation of humans and the ability to live with each other.

“UCLA institute on kindness wants to promote a more humane world” Comprehension Questions

Answer the following questions, citing evidence from the text.

- 1) In the “Mindfulness Can Change The Brain” section, which paragraph explains how mindfulness training improved someone’s life? Summarize the paragraph in the space below:

- 2) Which section of the text best explains what kindness entails? Write the name of the section below and include one sentence as evidence to support your claim.

Choose an argument or claim from the text, either made by the writer or by someone referred to by the writer. Summarize the argument or claim, and then explain why you agree or disagree. Support your position with evidence from the text.

The surprising differences between our view of the world and reality

By National Geographic, adapted by Newsela staff on 08.27.19

Word Count **832**

Level **1030L**



Cuban doctor Ralfis Carbort looks at a map of the world. Photo by: Ramon Espinosa/AP Photo

Most of us have a rough map of the world in our minds that we use any time we think about places. However, these mental maps aren't necessarily reliable. In fact, many of the maps in our heads share the same errors, some of which are quite large and surprisingly resistant to correction.

For instance, we all know that South America is south of North America. However, you may be surprised by the fact that virtually the entire South American continent is east of Florida.

Location, Location, Location

John Nelson is a cartographer, a maker of maps. He says there are lots of possible reasons for geographical errors like this one. Mental maps have to be simplified, and Nelson suspects the misplaced Americas may be partly a result of their names. After all, it's not called Southeast America.

Another commonly misplaced continent is Africa. North Americans tend to think of Africa as located almost entirely in the Southern Hemisphere, similar to South America. The truth is that around two-thirds of Africa is north of the equator. Africa actually extends northward to about the same latitude as Norfolk, Virginia.

Europe is also often placed much farther south on mental maps than it really is. People often think it is directly across the Atlantic Ocean from the mainland United States. It actually lines up better with Canada. Paris, France, is farther north than Montreal, Canada. Barcelona, Spain is at a similar latitude as Chicago, Illinois. Venice, Italy, lines up with Portland, Oregon.

Nelson suspects that climate might play a role in this mental mistake. Western Europe is relatively warm for its latitude thanks to the Gulf Stream, which brings warmer water from the Gulf of Mexico across the Atlantic. It gives Europe its so-called Mediterranean climate. The warmer temperatures are more similar to the climate of the lower 48 states than Canada's.

Inaccurate Perceptions Of Size

Locations aren't the only way our mental maps can be wrong. We also make mistakes about the relative size of things. This may be due in part to the nature of two-dimensional maps. Flattening a three-dimensional globe onto a flat surface isn't possible without some things appearing to move around. This is especially obvious for certain kinds of maps, such as the Mercator maps found in many 20th-century classrooms.

Mercator maps twist the shape and relative size of continents, particularly near the poles. This is why Greenland appears to be similar in size to all of South America on Mercator maps. In fact, South America is more than eight times larger than Greenland.

While you may have already been aware of Greenland's extreme exaggeration on some maps, other changes are more surprising. For example, Brazil doesn't seem that big on many maps, but really it's bigger than the entire continental United States and almost as large as Canada. Alaska, which is a giant on Mercator maps, is actually a little smaller than Libya in North Africa.

Mental Maps Are Simpler

Anthony Robinson is a geographer at Penn State University in Pennsylvania. He says thinking about the world in two dimensions also twists our ideas about how to get from one place to another. If you draw a line on a flat map from Washington, D.C., to Shanghai, China, the most direct route appears to be due west over the United States and the Pacific Ocean. However, flights to Asia usually fly over the North Pole. When looking at a globe instead of a flat map, it makes sense. "That's far and away the shortest way to get there," Robinson says.

None of these geographic mental mistakes would be surprising for a student to have on the first day of geography class. However, even once we've learned the truth, the errors on our mental maps tend to stick around. One reason for this may be that our ideas of the world are basically summaries of the geography, says cartographer Dylan Moriarty. The mental map is a simple model, similar to the the subway map of New York City, he says. Having a simpler idea in mind is "just more useful and practical," he says.

Robinson says our physical experience of the world may come into play as well. He says it probably has something to do with the limits of our ability to grasp such large amount of space. It also may have to do with new technology and methods of transportation that make it less important to memorize the details, he says.

Whatever the reason, these mental map errors are so common and stubborn that even professionals have them. A 1985 study of mental world maps found that geographers had the same misconceptions of the relative latitude of cities in North America and Europe as everyone else. This is true for cartographers, too. The "eastiness" of South America still surprises Nelson sometimes. "I really have to look again at a map and be like, is that really the case?" he says. "Sure enough, it is."

“The surprising differences between our view of the world and reality”
Comprehension Questions

Answer the following questions, citing evidence from the text.

- 1) Why do people think that the actual location of continents is different from their mental maps?

- 2) Why does flattening a three-dimensional globe onto a flat surface increase our mental map misperceptions?

What conclusions can you make about the importance of mental maps? Use at least two pieces of evidence from the article to support your conclusion.

Why Exercise Is Wise

You've probably heard countless times how exercise is "good for you." But did you know that it can actually help you feel good, too? Getting the right amount of exercise can rev up your energy levels and even help improve your mood.

Rewards and Benefits

Experts recommend that teens get 60 minutes or more of moderate to vigorous physical activity each day. Here are some of the reasons:

- **Exercise benefits every part of the body, including the mind.** Exercising causes the body to make chemicals that can help a person to feel good. Exercise can help people sleep better. It can also help some people who have mild depression and low self-esteem. Plus, exercise can give people a real sense of accomplishment and pride at having achieved a certain goal — like beating an old time in the 100-meter dash.
- **Exercise can help you look better.** People who exercise burn more calories and look more toned than those who don't. In fact, exercise can help keep your body at a healthy weight.
- **Exercise helps people lose weight and lower the risk of some diseases.** Exercising regularly decreases a person's risk of developing certain diseases, including obesity, type 2 diabetes, and high blood pressure.
- **Exercise can help a person age well.** This may not seem important now, but your body will thank you later. For example, osteoporosis (a weakening of the bones) can be a problem as people get older. Weight-bearing exercise — like jumping, running, or brisk walking — can help keep bones strong.

The three parts of a balanced exercise routine are: aerobic exercise, strength training, and flexibility training.

Aerobic Exercise

Like other muscles, the heart enjoys a good workout. Aerobic exercise is any type of exercise that gets the heart pumping and gets you breathing harder. When you give your heart and lungs this kind of workout regularly, they get stronger and are better at getting oxygen (in the form of oxygen-carrying blood cells) to all parts of your body.

If you play team sports, you're probably getting at least 60 minutes or more of moderate to vigorous activity on practice days. Some team sports that give you a great aerobic workout are basketball, soccer, lacrosse, hockey, and rowing.

But if you don't play team sports, don't worry — there are plenty of ways to get aerobic exercise. These include biking, running, swimming, dancing, in-line skating, tennis, cross-country skiing, hiking, and walking quickly.

Strength Training

The heart isn't the only muscle to benefit from regular exercise. The other muscles in your body enjoy exercise too. When you use your muscles, they become stronger.

Strong muscles are also a plus because they support your joints and help prevent injuries. Muscle also use more energy than fat does, so building your muscles will help you burn more calories and maintain a healthy weight.

You don't have to lift weights to make your muscles and bones stronger. Different types of exercise strengthen different muscle groups, for example:

- For arms, try rowing or cross-country skiing. Pull-ups and push-ups, those old gym class standbys, are also good for building arm muscles.
- For strong legs, try running, biking, rowing, or skating. Squats and leg raises also work the legs.
- For abdominal and core strength, you can't beat rowing, yoga or pilates, planks and crunches.

Flexibility Training

Strengthening the heart and other muscles isn't the only important goal of exercise. Exercise can also help the body stay flexible, meaning that your muscles and joints stretch and bend easily.

Being flexible may also help improve a person's sports performance. Some activities, like dance or martial arts, require great flexibility. But increased flexibility also can help people perform better at other sports, such as soccer or lacrosse.

Sports and activities that encourage flexibility are easy to find. Martial arts like karate, ballet, gymnastics, and yoga are good choices. Stretching after your workout will also help you improve your flexibility.

What's Right for Me?

One of the biggest reasons people drop an exercise program is lack of interest: If what you're doing isn't fun, it's hard to keep it up. The good news is there are tons of different sports and activities to try to see which one inspires you.

When picking the right type of exercise, it can help to consider your workout personality. For example, do you like to work out alone and on your own schedule? If so, solo sports like running, biking, or snowboarding could be for you. Or do you like the shared motivation and companionship that comes from being part of a team? School sports, intramural leagues, club teams, and pick-up games are great ways to stay active with others.

You also need to plan around practical considerations, such as whether your chosen activity is affordable and available to you. (Activities like horseback riding may be harder for people who live in cities, for example.) You'll also want to think about how much time you can set aside for your sport.

It's a good idea to talk to someone who understands the exercise, like a coach or fitness expert at a gym. He or she can get you started on a program that's right for you and your level of fitness.

Doctors know that most people benefit from regular exercise, even those with disabilities or medical problems like asthma. If you have a health problem or other concern (like being overweight or very out of shape), talk to your doctor before beginning an exercise plan.

Considering the benefits to the heart, muscles, joints, and mind, it's easy to see why exercise is wise. And the great thing about exercise is that it's never too late to start. Even small things can count as exercise when you're starting out — like taking a short bike ride, walking the dog, or raking leaves.

Reviewed by: Mary L. Gavin, MD
Date reviewed: January 2018

Note: All information on TeensHealth® is for educational purposes only. For specific medical advice, diagnoses, and treatment, consult your doctor.

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“Teens Health - Why Exercise Is Wise” Comprehension Questions

Answer the following questions, citing evidence from the text.

1) What are the three parts of a balanced exercise routine?

2) How does aerobic activity change the body?

Develop a week-long exercise routine using all three exercise components.

Scientists At Work: Diving in underwater caves for new forms of life

By Tom Iliffe, The Conversation, adapted by Newsela staff on 02.07.18

Word Count **900**

Level **1050L**



Image 1. Author Tom Iliffe (right) leads scientists in a cave dive. Photo by: Jill Heinerth

Maybe when you picture scientific research it involves experiments with test tubes and beakers. Or you imagine a university professor poring over old manuscripts in a library. But I also do something else – I go cave diving.

I study the biology and ecology of coastal caves and the marine life that inhabits them. I head underwater to explore these unique and challenging ecosystems. Often my cave-diving partners and I go to places no other human has been before. The peaks of the tallest mountains can be viewed from an airplane and the depths of the sea can be mapped with sonar. Caves, however, can only be explored in person.

Over the last 40 years, I have explored more than 1,500 underwater caves around the globe. I've explored caves from Australia to the Mediterranean, from Hawaii to the Bahamas and throughout the Caribbean. The experience can be breathtaking. When you are down 60 to 100 feet in a cave that has zero light and is 20 miles long, you never know what you are about to see.

My primary focus is searching for new forms of life that are specifically adapted to this dark environment with little food. The creatures I find are mostly white, eyeless crustaceans. Cave diving is necessary because the caves I'm interested in are filled with water. The only way to get to these areas is to strap on a scuba tank and jump in.

Scientific Research As Extreme Sport

The list of what can go wrong in a cave dive could fill your event planner. Some of these problems could be equipment or light failure, leaking scuba tanks, broken guide lines, getting lost, cave collapse, zero visibility or poisonous gas. This kind of fieldwork can be a matter of life or death. I have had some close calls over the years, and sadly, have lost several good friends and researchers in cave accidents.

Underwater caves can be very hostile and unforgiving. Fourteen people have died in the Devil's cave system in north-central Florida alone in the last 30

years. Most of the time, human error is to blame, because divers don't follow the rules or they lack essential training and experience.

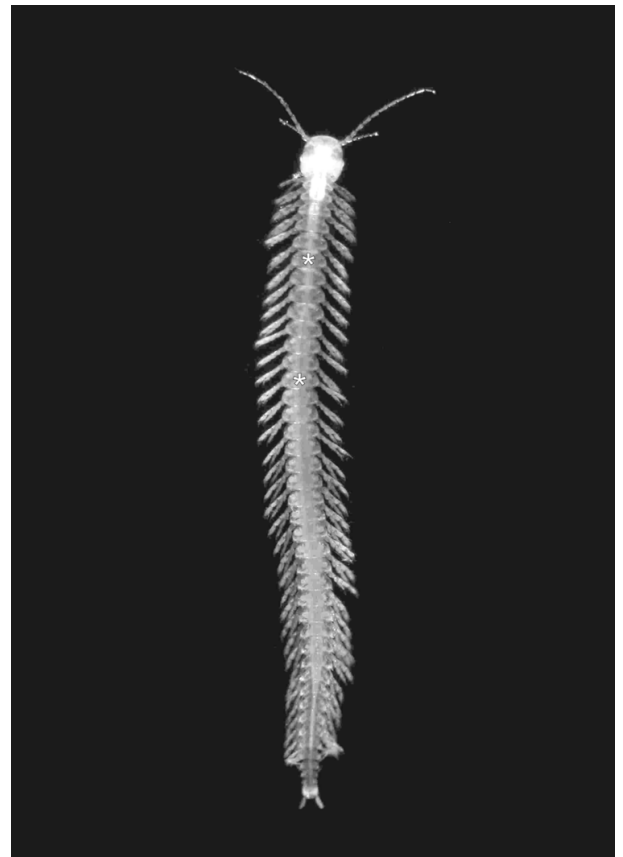
My family has gotten used to the idea that what I do is not always a walk in the park. They know that I stress safety and being physically and mentally prepared. I also religiously abide by the cardinal rule of cave diving – that you never, ever dive alone. My colleagues and I usually go into a cave with teams of other divers and constantly check on each other. Our dives usually last about 90 minutes, but can be as long as three hours or more.

Death-Defying Dives Pay Off In Discoveries

We are not just discovering new species. We are also finding higher groups of animals including classes, orders and families. Some of these animals have close relatives living in caves on opposite sides of the Atlantic Ocean or even the far side of Earth. Sometimes we make discoveries about geology as well. In Texas, our team discovered the deepest underwater cave in the U.S., which was 462 feet deep.

The graduate students in my lab work on many different questions. Some are uncovering the nature of chemosynthetic processes in caves and what it means for the cave food chain. In chemosynthesis, microorganisms use energy from chemical bonds, rather than energy from light as in photosynthesis.

Other students are examining cave sediment to understand sea levels during the Ice Age more than 10,000 years ago. Others are interested in the tree roots that penetrate underwater caves and their importance to the tropical forest above. We're finding evidence of sister species of cave animals on opposite sides of the Atlantic. They separated from one another about 110 million years ago, about when the Earth's plates moved apart to create the Atlantic Ocean. We're also

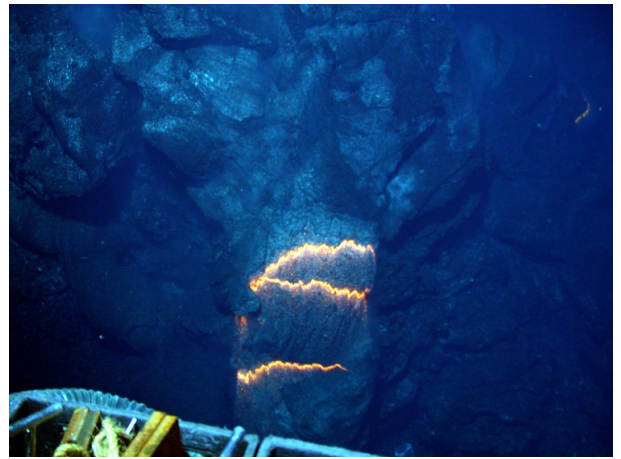


determining how environmental and ecological factors affect the availability and variety of animals in saltwater caves.

Our research has significant implications, especially for endangered species and environmental protection. Many cave animals can be found only in a single cave and nowhere else on Earth. Pollution or destruction of caves can cause the extinction of an entire species.

I am 69 years old, and to me the risks of cave diving are worth it. It's like the Star Trek mantra come true – to boldly go where no man has gone before. I have the chance to discover new forms of marine life, to view never-before-seen underwater formations, vast chambers, endless tunnels and deep chasms. I swim in some of the bluest and purest water on Earth. I will take that sort of research and its challenges any day. I love it, and I will tell you with all honesty that I can't wait until my next trip.

Tom Iliffe is a professor of marine biology at Texas A&M University in Texas.



“Scientists At Work: Diving in underwater caves for new forms of life” Comprehension Questions

Answer the following questions, citing evidence from the text.

- 1) What claim is the author making in this article? Underline three sentences from the article that support your answer.

- 2) What conclusions can you make about the pros and cons of underwater cave exploration?

How would physical damage to underwater caves possibly affect a species' population? Support your claim with evidence from the article. Then, explain why the evidence supports your claim.



Notes, Examples and Exam Questions

Learning Target: 2.4 To evaluate algebraic expressions, 2.5 To simplify algebraic expressions, 2.6 To create algebraic expressions.

Vocab:

- Terms: algebraic expressions separated by a + or a – sign
- Constant Term: a term that is a real number (no variable part)
- Coefficient: the numerical factor of a term
- Like Terms: terms that have the exact same variable part (must be the same letter raised to the same exponent) Note: Only *like terms* can be added or subtracted (combined).

▲ To add or subtract like terms, add or subtract the coefficients and keep the variable part.

Ex 1: Simplify $3\odot + 4\odot$.

This means we are adding 3 smiley faces plus 4 smiley faces. Therefore, we have a total of 7 smiley faces, which we can write as $7\odot$.

Ex 2: Use the expression $1 - 3x^2 + 4xy + x^2 + 5y^2$ to answer the following questions.

1. How many terms are in the expression? **Solution: There are 5 terms.**
2. Name the constant term(s). **Solution: There is one constant term, 1.**
3. How many like terms are in the expression?
Solution: There are 2 like terms. $-3x^2$ and x^2
4. What is the coefficient of the xy term? What is the coefficient of the fourth term?
Solution: The xy term has a coefficient of 4. The fourth term has a coefficient of 1.
5. Simplify the algebraic expression.
Solution: Combine like terms. $1 - 3x^2 + 1x^2 + 4xy + 5y^2 = 1 - 2x^2 + 4xy + 5y^2$
6. How many terms are in the simplified form of the algebraic expression?
Solution: There are 4 terms.



Ex 3: Translate the following phrases to algebraic expressions.

- | | | |
|----|---|-----------------|
| a. | Three more than five times a number | $5x + 3$ |
| b. | Twice a number decreased by seven | $2x - 7$ |
| c. | Twelve less than eight times a number | $8x - 12$ |
| d. | Twenty diminished by four times a number | $20 - 4y$ |
| e. | Fifty increased by three times a number | $50 + 3m$ |
| f. | The sum of triple a number and seventeen | $3w + 17$ |
| g. | Two times the difference of fifty and ten times a number | $2(50 - 10x)$ |
| h. | The quotient of fifteen and twice a number | $\frac{15}{2y}$ |
| i. | Eight less than the product of five times a number | $5x - 8$ |
| j. | Jim's age is 3 times Sarah's age | $3S$ |
| k. | The amount of money earned if you make \$7 per hour for x hours | $7x$ |



Variables and Expressions Worksheet

I. Indicate with math symbols what operations are being described by the given word(s). Use +, -, \times , or \div symbols.

- | | | |
|--------------------|-----------------------|--------------------------|
| 1. sum _____ | 2. product _____ | 3. decreased by _____ |
| 4. quotient _____ | 5. increased by _____ | 6. difference _____ |
| 7. more than _____ | 8. less than _____ | 9. twice something _____ |

II. Write a verbal expression for the algebraic expression.

- | | |
|-------------------|--------------------------|
| 10. ab | 11. $x + 7$ |
| 12. $2x$ | 13. m^3 |
| 14. $x - 6$ | 15. $8y^2$ |
| 16. $\frac{x}{y}$ | 17. $\frac{1}{2}(x + y)$ |
| 18. $3x - 4$ | 19. $5(a - b)$ |

III. Write an algebraic expression to the given verbal expression.

- | | |
|---|---|
| 20. eight less than a number | 21. a number increased by seven |
| 22. the quotient of m and n | 23. a number squared |
| 24. nine times a number | 25. a number decreased by three |
| 26. seven more than the cube of a number | 27. one-half the product of x and y |
| 28. the product of twice a and b | 29. twice the product of a and b |
| 30. two less than five times a number
number | 31. twice a number increased by three times the
number |



Notes, Examples and Exam Questions

Learning Target: 2.4 To evaluate algebraic expressions, 2.5 To simplify algebraic expressions, 2.6 To create algebraic expressions.

Simplifying an Algebraic Expression

Ex 1: Simplify the expression $4(x+8)-5x+1$.

Step One: Eliminate the parentheses using the distributive property.

$$\begin{aligned} 4x + 32 - 5x + 1 \\ = 4x - 5x + 32 + 1 \end{aligned}$$

Step Two: Combine like terms.

$$\begin{aligned} = -1x + 33 \\ = \boxed{-x + 33} \end{aligned}$$

Ex 2: Simplify the expression $5-2n(3n+8)-4n^2$

Step One: Eliminate the parentheses using the distributive property.

$$5 - 6n^2 - 16n - 4n^2$$

Note: The expression in parentheses is being multiplied by $-2n$, which is what was “distributed”.

Step Two: Combine like terms.

$$\begin{aligned} = 5 - 6n^2 - 4n^2 - 16n \\ = \boxed{5 - 10n^2 - 16n} \end{aligned}$$

Note: The answer may be written as $-10n^2 - 16n + 5$ by the commutative property.

Ex 3: Write a simplified expression for the perimeter of a rectangle with length $(x+7)$ and width $(x-2)$. Note: The formula for the perimeter of a rectangle is $P = 2l + 2w$.

Step One: Substitute the length and width into the formula.

$$\begin{aligned} 2(x+7) + 2(x-2) \\ = 2x + 14 + 2x - 4 \end{aligned}$$

Step Two: Simplify using the distributive property, combine like terms.

$$= \boxed{4x + 10}$$



Unit 2 - Expressions

SIMPLIFYING EXPRESSIONS WORKSHEET

1) $3(5x - 4)$

2) $-11(3 + 3x)$

3) $(15 - 3x)(-4)$

4) $15x + 3(-2x - 8)$

5) $15 + 3(-2x - 8)$

6) $5(2x + 3) - 8(4x - 12)$

7) $2x + 4(3x - 5) - 8$

8) $47x - 52x + 3(2x - 5) - 18$

9) $3.23 + 4x - 5(3x - 1.11)$

10) $10(3x - 5) - 7x + 3(2x + 4)$

11) A music store is receiving a shipment of I-pods. Each box comes with 6 red, 6 orange, 6 blue, 6 pink, 12 white, 12 black and 24 grey I-pods. If the store is receiving 8 boxes, how many of each color I-pod will they have?

12) A music store is receiving a shipment of I-pods. Each box comes with 12 white, 12 black and 24 grey I-pods. They already have 23 white, 16 black and 22 grey I-pods. If the store is receiving 5 boxes, how many of each color I-pod will they have?



EXPRESSIONS

Notes, Examples and Exam Questions

Learning Target: 2.4 To evaluate algebraic expressions, 2.5 To simplify algebraic expressions, 2.6 To create algebraic expressions.

Vocab:

- Terms: algebraic expressions separated by a + or a – sign
- Constant Term: a term that is a real number (no variable part)
- Coefficient: the numerical factor of a term
- Like Terms: terms that have the exact same variable part (must be the same letter raised to the same exponent) Note: Only *like terms* can be added or subtracted (combined).

▲ To add or subtract like terms, add or subtract the coefficients and keep the variable part.

Ex 1: Simplify $3\odot + 4\odot$.

This means we are adding 3 smiley faces plus 4 smiley faces. Therefore, we have a total of 7 smiley faces, which we can write as $7\odot$.

Ex 2: Use the expression $1 - 3x^2 + 4xy + x^2 + 5y^2$ to answer the following questions.

1. How many terms are in the expression? **Solution: There are 5 terms.**
2. Name the constant term(s). **Solution: There is one constant term, 1.**
3. How many like terms are in the expression?
Solution: There are 2 like terms. $-3x^2$ and x^2
4. What is the coefficient of the xy term? What is the coefficient of the fourth term?
Solution: The xy term has a coefficient of 4. The fourth term has a coefficient of 1.
5. Simplify the algebraic expression.
Solution: Combine like terms. $1 - 3x^2 + 1x^2 + 4xy + 5y^2 = 1 - 2x^2 + 4xy + 5y^2$
6. How many terms are in the simplified form of the algebraic expression?
Solution: There are 4 terms.



Evaluating an Algebraic Expression

Ex 3: Evaluate the expression $2x^3 - x^2 + y$ when $y = 2$ and $x = -3$.

Step One: Substitute in the values	$2(-3)^3 - (-3)^2 + 2$
	$2(-27) - (9) + 2$
Step Two: Perform the indicated operations	$-54 - 9 + 2$
	$= \boxed{-61}$

****Note:** Make sure to stress the importance of using parentheses when substituting values into the expression.

Ex 4: Evaluate the expression $x^2 - x^3y + 6$ when $y = 9$ and $x = 2$.

Step One: Substitute in the values	$= (2)^2 - (2)^3 \cdot (9) + 6$
	$= 4 - 8 \cdot 9 + 6$
Step Two: Perform the indicated operations	$= \boxed{-62}$

Simplifying an Algebraic Expression

Ex 5: Simplify the expression $4(x+8) - 5x + 1$.

Step One: Eliminate the parentheses using the distributive property.	$4x + 32 - 5x + 1$
	$= 4x - 5x + 32 + 1$
Step Two: Combine like terms.	$= -1x + 33$
	$= \boxed{-x + 33}$

Ex 6: Simplify the expression $5 - 2n(3n+8) - 4n^2$

Step One: Eliminate the parentheses using the distributive property. $5 - 6n^2 - 16n - 4n^2$
 Note: The expression in parentheses is being multiplied by $-2n$, which is what was “distributed”.

	$= 5 - 6n^2 - 4n^2 - 16n$
Step Two: Combine like terms.	$= \boxed{5 - 10n^2 - 16n}$

Note: The answer may be written as $-10n^2 - 16n + 5$ by the commutative property.

Ex 7: Write a simplified expression for the perimeter of a rectangle with length $(x+7)$ and width $(x-2)$. Note: The formula for the perimeter of a rectangle is $P = 2l + 2w$.

Step One: Substitute the length and width into the formula.	$2(x+7) + 2(x-2)$
---	-------------------



Step Two: Simplify using the distributive property, combine like terms.

$$= 2x + 14 + 2x - 4$$

$$= \boxed{4x + 10}$$

Ex 8: Translate the following phrases to expressions.

- | | | |
|----|---|-----------------|
| a. | Three more than five times a number | $5x + 3$ |
| b. | Twice a number decreased by seven | $2x - 7$ |
| c. | Twelve less than eight times a number | $8x - 12$ |
| d. | Twenty diminished by four times a number | $20 - 4y$ |
| e. | Fifty increased by three times a number | $50 + 3m$ |
| f. | The sum of triple a number and seventeen | $3w + 17$ |
| g. | Two times the difference of fifty and ten times a number | $2(50 - 10x)$ |
| h. | The quotient of fifteen and twice a number | $\frac{15}{2y}$ |
| i. | Eight less than the product of five times a number | $5x - 8$ |
| j. | Jim's age is 3 times Sarah's age | $3S$ |
| k. | The amount of money earned if you make \$7 per hour for x hours | $7x$ |



Unit 2 - Expressions

Working With Expressions Worksheet

Match the vocabulary to the correct definition. Write the answer in the blank on the left side of that paper.

- | | |
|-------------------------------|--|
| _____ 1. Algebraic Expression | A. Each part of an expression separated by an operation |
| _____ 2. Coefficient | B. A number that stands by itself |
| _____ 3. Constant | C. A number that does not stand by itself. It is attached to the variable. |
| _____ 4. Term | D. A letter that stands for a particular numerical value |
| _____ 5. Variable | E. A number sentence without an equal sign, has at least one two terms and one operation |

On 1 and 2, identify each part of the algebraic expression as the term, coefficient, constant, or variable.	
1. $4x - 12$ 4 is a(n) _____ x is a(n) _____ 4x is a(n) _____ -12 is a(n) _____	2. $a + 3b$ a is a(n) _____ 3 is a(n) _____ 3b is a(n) _____
3. Simplify $-8(-5b + 7) + 8b$	4. Simplify $-4p - (1 - 6p)$
5. Simplify $8p - 3 + 9p + 6p + 2p - 6p + 8 - 3p + 9 - p - 12$	6. Simplify $x(3y - 2) + y(2x + 5) - 8xy + 12x - 2y$

DIRECTIONS: Translate each of the following into an equation.

1. Three less than "x" is equal to 13. _____
2. The product of 9 and "m" is 45. _____



Unit 2 - Expressions

3. A number divided by 6 is 18.

4. A number plus 17 is 25.

5. Seven times a number is 28.

6. A number divided by 7 is 9.

7. A number minus 12 is 20.

8. The quotient of "y" and 3 is 25.

9. One-fifth of "r" is 15.

10. Six less than 2 times "y" is 34.

11. Five more than "n" is equal to 9.

12. A number increased by 3 is 19.

13. The difference of "p" and 7 is 30.

14. Fifteen multiplied by "k" is 75.

15. The sum of $3y$ and 5 is 47.



Notes and Examples

Solving One-Step Equations:

Ex 1: Solve the equation $x + 6 = -8$.

Because 6 is being **added** to x , we will use the inverse operation (**subtraction**) to isolate the variable.

$$\begin{array}{l} \text{Use the Subtraction Property of Equality} \\ x + 6 = -8 \\ \quad -6 \quad -6 \\ \hline x = -14 \\ \\ x + 6 = -8 \\ \text{Check your solution in the original equation: } (-14) + 6 = -8 \\ -8 = -8 \end{array}$$

Ex 2: Solve the equation $-4 = x - 3$.

Because 3 is being **subtracted** from x , we will use the inverse operation (**addition**) to isolate the variable.

$$\begin{array}{l} \text{Use the Addition Property of Equality} \\ -4 = n - 3 \\ \quad +3 \quad +3 \\ \hline -1 = n \\ \\ \text{You can write the solution as } n = -1 \text{ by the symmetric property.} \\ \\ -4 = n - 3 \\ \text{Check your solution in the original equation: } -4 = (-1) - 3 \\ -4 = -4 \end{array}$$



Ex 3: Solve the equation $15x = 3$.

Because x is being **multiplied** by 15, we will use the inverse operation (**division**) to isolate the variable.

$$\frac{15x}{15} = \frac{3}{15}$$

Use the Division Property of Equality

$$x = \frac{3}{15}$$

$$x = \frac{1}{5}$$

$$15x = 3$$

Check your solution in the original equation:

$$15\left(\frac{1}{5}\right) = 3$$

$$3 = 3$$

Ex 5: Solve the equation: $\frac{t}{-2} = 9$.

Because t is being **divided** by -2 , we will use the inverse operation (**multiplication**) to isolate the variable.

$$\frac{t}{-2} = 9$$

Use the Multiplication Property of Equality

$$-2 \cdot \frac{t}{-2} = -2 \cdot 9$$

$$t = -18$$

$$\frac{t}{-2} = 9$$

Check your solution in the original equation:

$$\frac{(-18)}{-2} = 9$$

$$9 = 9$$


Solving an Equation in the Form: $ax + b = c$

▲ Note: To solve an equation, we use the properties of equality to isolate the variable on one side of the equation. When isolating the variable, we will “undo” **addition/subtraction first**, then “undo” multiplication/division using inverse operations. ****This is the order of operations in reverse!**

Ex 1: Solve the equation $-8x + 6 = 5$.

	$-8x + 6 = 5$	
Step One: Undo addition (using subtraction).	$\begin{array}{r} -6 \quad -6 \\ -8x \quad = -1 \end{array}$	
	$\frac{-8x}{-8} = \frac{-1}{-8}$	
Step Two: Undo multiplication (using division).	$\boxed{x = \frac{1}{8}}$	
	$-8x + 6 = 5$	
Step Three: Check your solution.	$\begin{array}{l} -8\left(\frac{1}{8}\right) + 6 = 5 \\ \Rightarrow \end{array}$	$\begin{array}{l} -1 + 6 = 5 \\ 5 = 5 \end{array}$

Ex 2: Solve the equation $-4 = -8 + 3x$.

Step One: Rewrite in the form $ax + b = c$.	Symmetric Property of Equality: $-8 + 3x = -4$	
	Commutative Property of Addition: $3x - 8 = -4$	
	$3x - 8 = -4$	
Step Two: Undo subtraction (using addition).	$\begin{array}{r} +8 \quad +8 \\ 3x \quad = 4 \end{array}$	
	$\frac{3x}{3} = \frac{4}{3}$	
Step Three: Undo multiplication (using division).	$\boxed{x = \frac{4}{3}}$	
	$-4 = -8 + 3x$	
Step Four: Check your solution.	$\begin{array}{l} -4 = -8 + 3\left(\frac{4}{3}\right) \\ \Rightarrow \end{array}$	$\begin{array}{l} -4 = -8 + 4 \\ -4 = -4 \end{array}$

Unit 2 – Equations in One Variable
SOLVING ONE AND TWO-STEP EQUATIONS WORKSHEET



1) $w - 17 = 28$

2) $-18 + y = 93$

3) $-5d = 155$

4) $\frac{x}{12} = 13$

5) $3m - 11 = 22$

6) $17 = -7p + 38$

7) $\frac{3}{4}x - 16 = 8$

8) $\frac{x}{8} + 19 = 11$

9) $7.5 = 2r - 1.5$

10) $14 + \frac{s}{3} = 19$

11) $6 = -12 + \frac{h}{-7}$

12) $5 - 9w = 23$



- 13) When solving for x in the equation $-7 + 4x = 42$, the first step is to _____
- 14) Mr. Becker's class is told to isolate the variable in the equation $13 = -7x - 48$. Which student follows the correct steps?
- a) Kevin subtracts 13 and then divides by -7.
 - b) Jorge adds 48 and then adds 7.
 - c) Gabriella divides by -7 and then adds 48.
 - d) Jayson adds 48 and then divides by -7.
- 15) What is the inverse operation of division? _____
- 16) The length of a table top is twice its width. The area of the table top is 18 ft^2 . Find the length and width of the table.
- 17) Felicia has \$4.17 in change. After she takes out the quarters, she has \$1.42 left. How many quarters did she take out?



Solving an Equation in the Form: $ax + b = c$

▲ Note: To solve an equation, we use the properties of equality to isolate the variable on one side of the equation. When isolating the variable, we will “undo” **addition/subtraction first**, then “undo” multiplication/division using inverse operations. ****This is the order of operations in reverse!**

Ex 1: Solve the equation $-8x + 6 = 5$.

$$\begin{array}{lcl}
 & -8x + 6 = 5 & \\
 \text{Step One: Undo addition (using subtraction).} & -6 & -6 \\
 & -8x & = -1 \\
 & \frac{-8x}{-8} = \frac{-1}{-8} & \\
 \text{Step Two: Undo multiplication (using division).} & & \boxed{x = \frac{1}{8}}
 \end{array}$$

$$\begin{array}{lcl}
 & -8x + 6 = 5 & \\
 \text{Step Three: Check your solution.} & -8\left(\frac{1}{8}\right) + 6 = 5 & \Rightarrow \begin{array}{l} -1 + 6 = 5 \\ 5 = 5 \end{array}
 \end{array}$$

Ex 2: Solve the equation $-4 = -8 + 3x$.

$$\begin{array}{lcl}
 \text{Step One: Rewrite in the form } ax + b = c. & \text{Symmetric Property of Equality: } -8 + 3x = -4 & \\
 & \text{Commutative Property of Addition: } 3x - 8 = -4 & \\
 & 3x - 8 = -4 & \\
 \text{Step Two: Undo subtraction (using addition).} & +8 & +8 \\
 & 3x & = 4 \\
 & \frac{3x}{3} = \frac{4}{3} & \\
 \text{Step Three: Undo multiplication (using division).} & & \boxed{x = \frac{4}{3}}
 \end{array}$$

$$\begin{array}{lcl}
 & -4 = -8 + 3x & \\
 \text{Step Four: Check your solution.} & -4 = -8 + 3\left(\frac{4}{3}\right) & \Rightarrow \begin{array}{l} -4 = -8 + 4 \\ -4 = -4 \end{array}
 \end{array}$$



▲ For more complicated equations, you may have to **simplify both sides of the equation** **first** using the distributive property and combining like terms. Note: Our goal is to write the equation in $ax + b = c$ form.

Ex 3: Solve the equation: $6(x - 4) - 2x = 24$

Step One: Use the distributive property.

$$6x - 24 - 2x = 24$$

Step Two: Combine like terms.

$$4x - 24 = 24 \quad (\text{This is in } ax + b = c \text{ form.})$$

Step Three: Undo subtraction (using addition).

$$4x - 24 = 24$$

$$+ 24 \quad + 24$$

$$4x = 48$$

Step Four: Undo multiplication (using division).

$$\frac{4x}{4} = \frac{48}{4}$$

$$x = 12$$

Step Five: Check your solution.

$$6(x - 4) - 2x = 24$$

$$6((12) - 4) - 2(12) = 24$$

$$6(8) - 2(12) = 24$$

$$\Rightarrow \begin{aligned} 48 - 24 &= 24 \\ 24 &= 24 \end{aligned}$$

Ex 4: Solve the equation: $20 = 5y - 3(4 - y)$

Step One: Use the distributive property.

$$20 = 5y - 12 + 3y$$

Step Two: Combine like terms.

$$20 = 8y - 12$$

▲ Note: We will write this in $ax + b = c$ form using the symmetric property. $8y - 12 = 20$

Step Three: Undo subtraction (using addition).

$$8y - 12 = 20$$

$$+ 12 \quad + 12$$

$$8y = 32$$

Step Four: Undo multiplication (using division).

$$\frac{8y}{8} = \frac{32}{8}$$

$$y = 4$$

Step Five: Check your solution.

$$20 = 5y - 3(4 - y)$$

$$20 = 5(4) - 3(4 - (4))$$

$$20 = 20 - 3(0)$$

$$\Rightarrow \begin{aligned} 20 &= 20 - 0 \\ 20 &= 20 \end{aligned}$$



Solve a Linear Equation with Variables on Both Sides

▲ If there are variables on both sides, you will need to first simplify both sides of the equation and then bring all variables to the same side using inverse operations.

Ex 15: Solve the equation $3x - 2 = 4(x + 6)$

Step One: Use the distributive property.

$$3x - 2 = 4x + 24$$

$$3x - 2 = 4x + 24$$

Step Two: Bring all variables to the same side.

$$-4x \quad -4x$$

$$-1x - 2 = 24$$

Note: This is now in $ax + b = c$ form.

$$-1x - 2 = 24$$

Step Three: Undo subtraction (using addition).

$$+2 \quad +2$$

$$-1x = 26$$

Step Four: Undo multiplication (using division).

$$\frac{-1x}{-1} = \frac{26}{-1}$$

$$x = -26$$

Step Five: Check your solution.

$$3x - 2 = 4(x + 6)$$

$$3(-26) - 2 = 4((-26) + 6) \Rightarrow$$

$$-78 - 2 = 4(-20)$$

$$-80 = -80$$

Solve Linear Equations with Fractions or Decimals

When solving an equation with **fractions**, it saves time to “wipe-out” (or clear) the fractions by multiplying both sides by the LCD (lowest common denominator). This means rewriting an equivalent equation with integer terms and coefficients.

Ex 16: Solve the equation: $\frac{3}{5}x - \frac{9}{10} = \frac{1}{3} + 2x$

The LCD of the fractions is 30. Multiply every term in the equation by 30:

$$30 \cdot \frac{3}{5}x - 30 \cdot \frac{9}{10} = 30 \cdot \frac{1}{3} + 30 \cdot 2x \quad \Rightarrow \quad 18x - 27 = 10 + 60x$$



Now use the steps illustrated in the previous examples to solve the equation.

When solving an equation with **decimals**, it saves time to “wipe-out” (or clear) the decimals by multiplying both sides by a power of 10. This means rewriting an equivalent equation with integer terms and coefficients.

Ex 17: Solve the equation: $3.2 + 5x = 4.3x - 1.7$

This equation has decimals with digits in the tenth place. Therefore, we will multiply both sides 10 to wipe out the decimals.

$$10 \cdot 3.2 + 10 \cdot 5x = 10 \cdot 4.3x - 10 \cdot 1.7 \quad \Rightarrow \quad 32 + 50x = 43x - 17$$

Now use the steps illustrated in the previous examples to solve the equation.



Ex 18: Solve the equation: $4.23 = 5.8x - 6$

This equation has decimals with digits in the hundredths place. Therefore, we will multiply both sides by 100 to wipe out the decimals.

$$100 \cdot 4.23 = 100 \cdot 5.8x - 100 \cdot 6 \quad \Rightarrow \quad 423 = 580x - 600$$

$$\begin{array}{rcll} & 423 = 580x - 600 & & \\ \text{Isolate the variable:} & +600 & +600 & \Rightarrow \\ & 1023 = 580x & & \frac{1023}{580} = \frac{580x}{580} \\ & & & \boxed{1.76 \approx x} \end{array}$$

Note: Using the calculator, we rounded (approximated) the solution. Therefore, we use the symbol for APPROXIMATELY EQUAL TO: \approx



MULTI-STEP EQUATIONS WORKSHEET

Solve each equation. Then check your solution.

1. $17 + 2n = 21 + 2n$

2. $5x + 4 - 7x = 5 + x - 13$

3. $\frac{3}{4}n + 16 = 2 - \frac{1}{8}n$

4. $18 - 3.8x = 7.36 - 1.9x$

5. $4(x - 2) = 4x$

6. $1.03x - 4 = -2.15x + 8.72$



7. $6 = 3 + 5(y - 2)$

8. $5 - \frac{1}{2}(b - 6) = 4$

9. $2(x - 3) + 5 = 3(x - 1)$

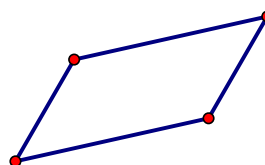
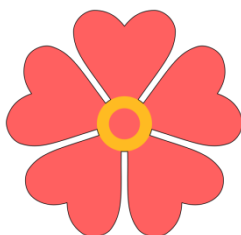
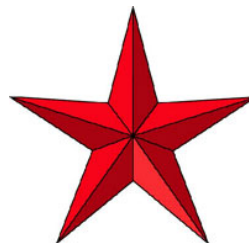
10. $-3(2n - 5) = \frac{1}{2}(-12n + 30)$

11. $2(x - 7) + 3(x - 1) = 4x + 3$

12. $9x - 4 = 4(x - 1) - 5x + 4$

Rotational symmetry is when a shape maps onto itself using a rotation. Rotational symmetry requires an order (number of times it maps onto itself in 360°) and an angle (first angle size to map onto itself).

1) Draw in the lines of symmetry for each of the shapes. If none, leave the diagram blank.



(Parallelogram)



2) Use the diagrams from question #1 to determine the order and angle of rotation symmetry for the following shapes. If none, write none.

Angle = _____ Angle = _____ Angle = _____ Angle = _____

Angle = _____ Angle = _____ Angle = _____ Angle = _____

3) Draw a figure that meets the given symmetry requirements. It must have:

rotational symmetry.	line symmetry.	symmetry.

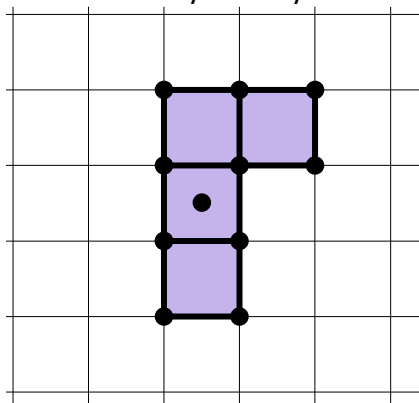


4) a) Draw three different figures, each having exactly one line of symmetry.

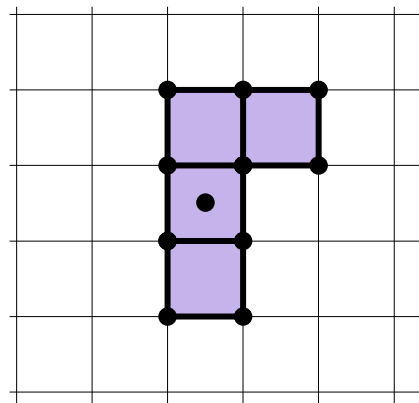
b) Do you notice any similarities in these three shapes?

5) Add blocks to the diagram to form the requirements.

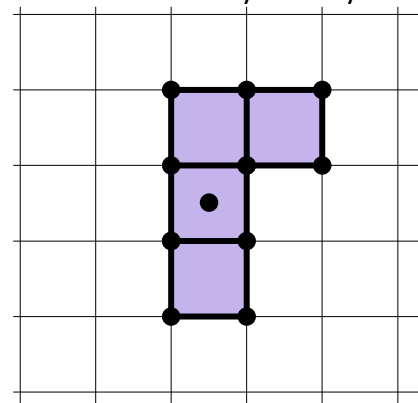
a) Rotational symmetry order of 2 with NO line symmetry.



b) Rotational symmetry order of 4.



c) Rotational symmetry of order 2 and two lines of symmetry.



6) Here are the letters of the alphabet. Classify them into the given categories.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

One Line of Symmetry	Two Lines of Symmetry	Rotational Symmetry
Line & Rotational Symmetry	No Symmetry	

**OBJECTIVE #:** G.CO.4**OBJECTIVE**

Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular line, parallel lines, and line segment.

BIG IDEA (Why is this included in the curriculum?)

- All two dimensional geometric figure can be created by transformations. Congruency and similarity may be proven by one or more transformation(s) on the pre-image.

PREVIOUS KNOWLEDGE (What skills do they need to have to succeed?)

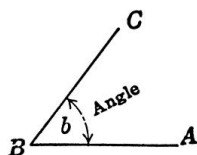
- The student must have a thorough knowledge of all types of angles.
- The student must also understand a complete rotation is 360° , as it relates to circles.
- The student must understand the properties of parallel and perpendicular lines.
- The student must know how to find the length of a line segment using the distance formula.
- The student must know how to find the slope of a line.

VOCABULARY USED IN THIS OBJECTIVE (What terms will be essential to understand?)**PREVIOUS VOCABULARY** (Terms used but defined earlier)

- Angle of Rotation: The angle formed when rays are drawn from the center of rotation to a point and to its image.
- Center of Rotation: A fixed point around which a figure is rotated.
- Circle: The set of all points in a plane that are equidistant from a given point, called the center.
- Initial Point: The starting point of a ray or vector.
- Line Segment: A portion of a line that consists of two endpoints and all points in between the two endpoints.
- Negative Rotation: A clockwise rotation.
- Parallel Lines: Two lines that are coplanar and do not intersect.
- Perpendicular: Two lines/segments/rays that intersect to form right angles.
- Positive Rotation: A counterclockwise rotation.
- Reflection: A rigid transformation in which the image is a mirror image of the pre-image, thus ensuring the pre-image and the image are equidistant from the line of reflection.
- Rotation: A rigid transformation that turns a figure about a fixed point, thus ensuring the pre-image and image are congruent.
- Translation: A rigid transformation that slides an object a fixed distance in a given direction, thus ensuring the pre-image and image are congruent.

NEW VOCABULARY (New Terms and definitions introduced in this objective)

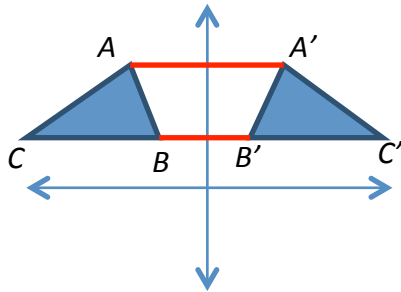
- Angle: A geometric figure formed by rotating a ray about its initial point. [G.CO.7, G.CO.8, G.CO.9]



Notation: $\angle B$, $\angle ABC$, $\angle CBA$

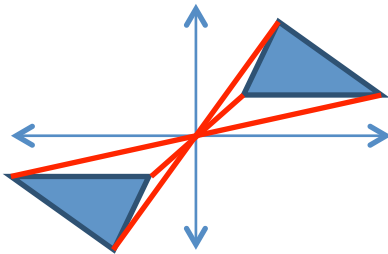


Reflections:



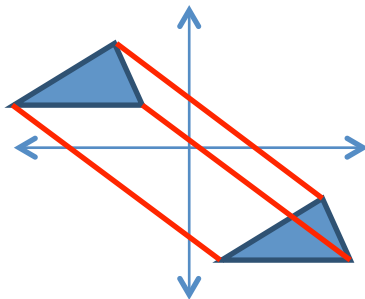
- Isometric – Within the shape distances, angle measures, parallelism, collinearity are all preserved.
- Orientation is reversed.

Rotations:



- Isometric – Within the shape distances, angle measures, parallelism, collinearity are all preserved.
- Orientation is preserved.

Translations:

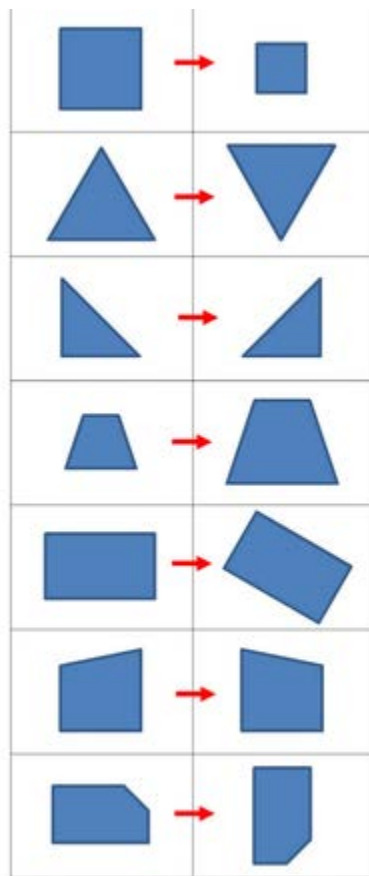


- Isometric – Within the shape distances, angle measures, parallelism, collinearity are all preserved.
- Orientation is preserved.

- In order to help student understand the rules for different transformations, it is useful to provide each student with a coordinate grid. Have students pick points on the coordinate plane to create a polygon. Allow students to work in groups to determine the rules for the following transformations

○ Reflection over the y -axis	$R_{x\text{-axis}}(x, y) = (______)$	$(x, -y)$
○ Reflection over the x -axis	$R_{y\text{-axis}}(x, y) = (______)$	$(-x, y)$
○ Rotation by 90° about the origin	$R_{O,90^\circ}(x, y) = (______)$	$(-y, x)$
○ Rotation by 180° about the origin	$R_{O,180^\circ}(x, y) = (______)$	$(-x, -y)$
○ Rotation by 270° (-90°) about the origin	$R_{O,270^\circ}(x, y) = (______)$	$(y, -x)$
○ Reflection over a vertical line $x = c$	$R_{x=c}(x, y) = (______)$	$(-x + 2c, y)$
○ Reflection over a horizontal line $y = b$	$R_{y=b}(x, y) = (______)$	$(x, -y + 2b)$
○ Reflection over the $y = x$ line	$R_{y=x}(x, y) = (______)$	(y, x)
○ Reflection over the $y = -x$ line	$R_{y=-x}(x, y) = (______)$	$(-y, -x)$

- 1) Circle the word that describes how the 1st shape transforms to the 2nd shape.



Resized

Flipped

Turned

Resized

Flipped

Turned

Resized

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Turned

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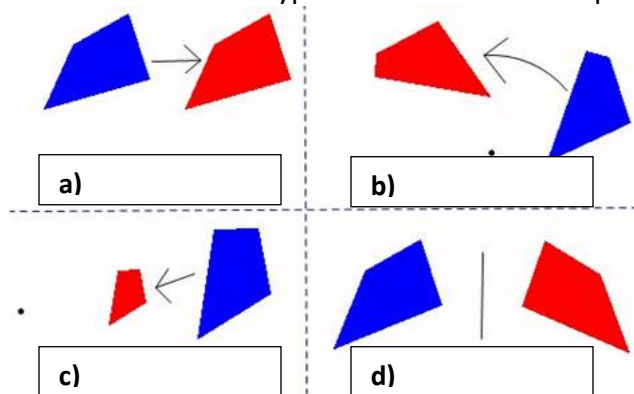
Turned

Resized

Flipped

Turned

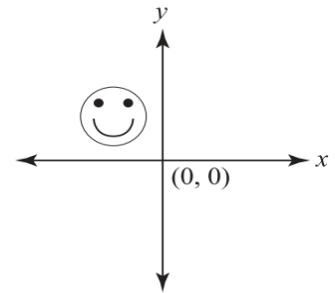
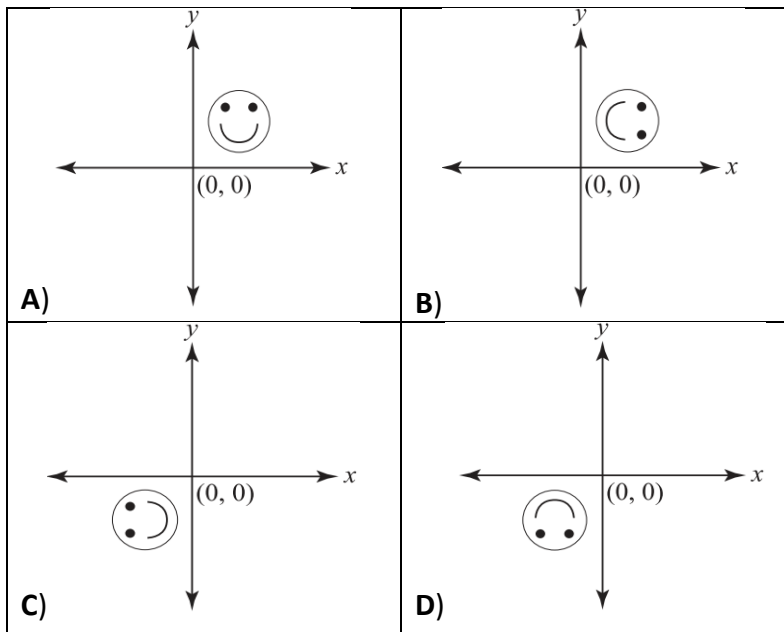
- 2) Name the different types of transformations pictured below.



- 3) Explain how to tell which figure is the pre-image and which is the image.
- 4) How are dilations different from translations and reflections?

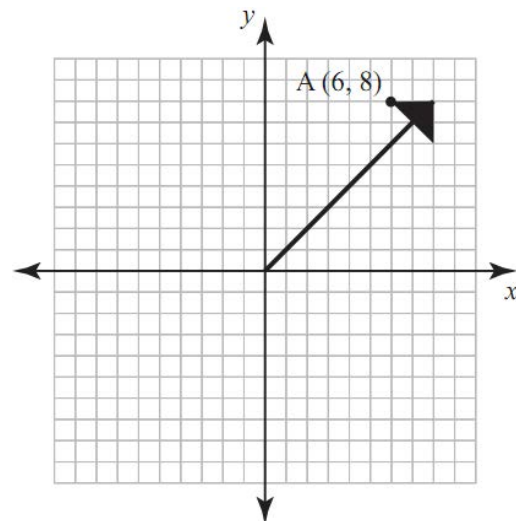


- 5) What would the figure at the right look like if it were reflected over the x -axis?

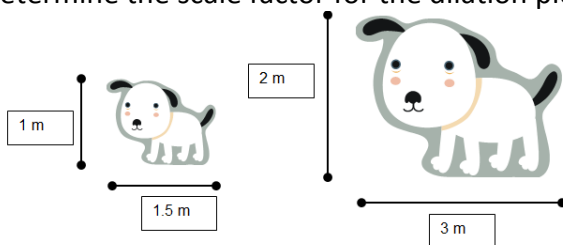


- 6) The arrow below represents the needle on a compass. The needle is rotated 180° in the clockwise direction. What are the coordinates of point A after the rotation?

- A) $(-8, -6)$
 B) $(-8, 6)$
 C) $(-6, -8)$
 D) $(6, -8)$

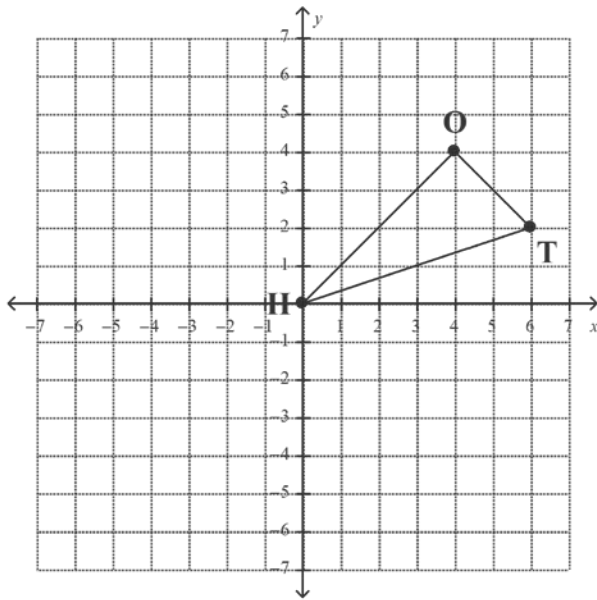


- 7) Determine the scale factor for the dilation pictured below:





- 8) Draw the translation of the triangle HOT six units left and one unit down. Label the image $H'O'T'$. Is the image similar or congruent? How do you know?



- 9) For the coordinate (x, y) , match the operation with the output:

- | | | |
|--|-------|------------------|
| a. Shift up 4 units; shift left 3 units | _____ | $(x, -y)$ |
| b. Rotate clockwise 90° | _____ | $(-x, -y)$ |
| c. Reflect over the x-axis | _____ | $(x + 4, y - 3)$ |
| d. Shift down 3 units; shift right 4 units | _____ | $(x - 3, y + 4)$ |
| e. Reflect over the y-axis | _____ | $(y, -x)$ |
| f. Reflect over the origin | _____ | $(-x, y)$ |

(Hint: pick a random point and try each operation)

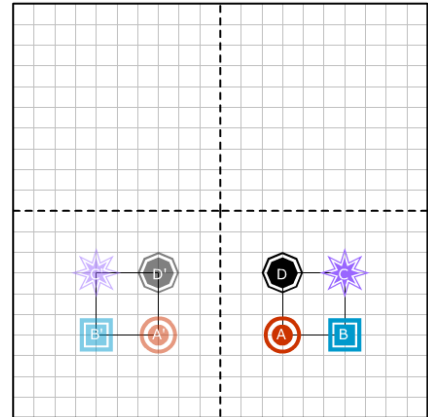
Geometry – Unit 2 Practice
Recognizing Transformations

G.CO.A.2, G.CO.A.4

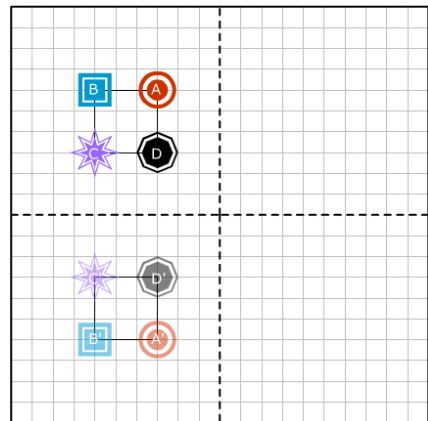
Name: _____!

Date: _____ Pd: _____

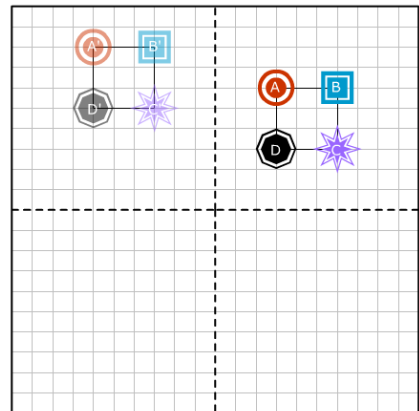
1) This shape has been reflected over the _____ axis.



2) This shape has been reflected over the _____ axis.

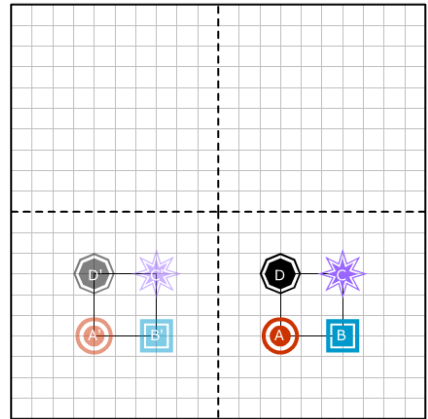


3) This shape has been translated _____ units _____, and _____ units _____.
 (Left/Right) (Up/Down)



SNRPDP

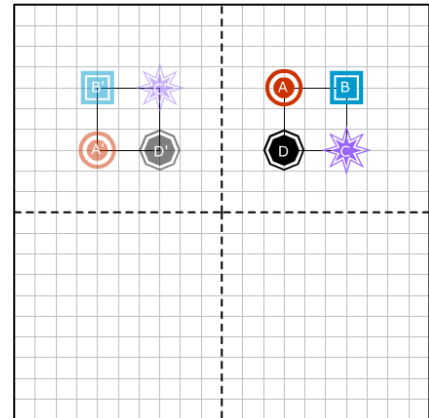
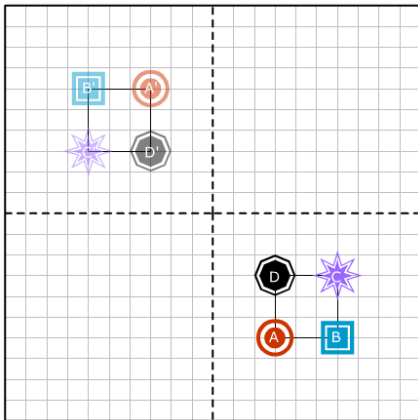
- 4) This shape has been translated ____ units _____, and ____ units _____.
 (Left/Right) (Up/Down)



- 5) Choose a matching rotation for the following:

- a) What is the same as a 90° clockwise rotation? _____
 b) What is the same as a 180° clockwise rotation? _____
 c) What is the same as a 270° clockwise rotation? _____

- 6) a) This shape has been rotated ____ degrees clockwise or ____ degrees counterclockwise.



- b) This shape has been rotated ____ degrees clockwise or ____ degrees counterclockwise.

- 7) State the coordinates of the image given the following transformation of point $P(-5, 4)$:

- a) reflected over the x-axis _____.
 b) translated $\langle -3, 5 \rangle$ _____.
 c) rotated 180° about the origin _____.

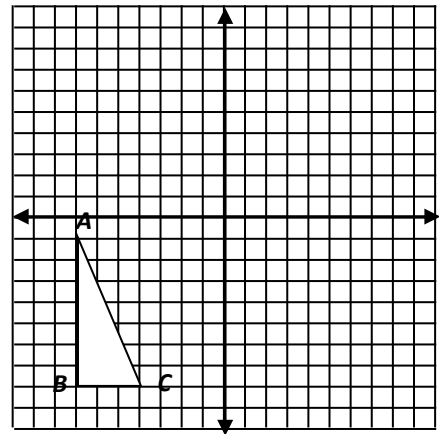
- 8) Find some patterns in your environment that were created using glide reflections. Make a sketch of the pattern.

OR

- 9) Use glide reflections to create your own patterns.



- 10) Translate $\triangle ABC$, 7 units up and 2 units right. Reflect the translated image over the y -axis. Draw $\triangle A'B'C'$



SAT Multiple Choice Problems:

- 11) Which figure could be a translation of R ?

A.

B.

C.

D.

- 12) What are the coordinates of the image of $(5, 0)$ under a rotation of 90° clockwise about the origin?

A. $(-5, 0)$

B. $(0, 5)$

C. $(0, -5)$

D. $(5, -5)$

- 13) If $\triangle ABC$ with vertices $A(-2, 0)$, $B(-2, -2)$, $C(-4, -2)$ is reflected over the x -axis and then the image is reflected over the y -axis, which coordinates would represent the final image of A ?

A. $(-2, 0)$

B. $(2, 0)$

C. $(0, 2)$

D. $(0, -2)$

Quick Concept: An **Isometry** is a transformation in the plane that preserves distance, angle measure, shape, and size..... all things about the shape. In other words, **an isometric transformation produces an identical (or congruent) shape.**

1) Circle which of the following are isometric transformations.

Pre-Image



a)



b)



c)



d)



e)

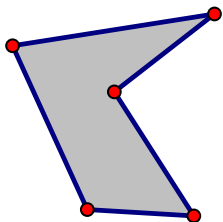


f)

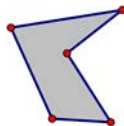


2) Circle which of the following are isometric transformations.

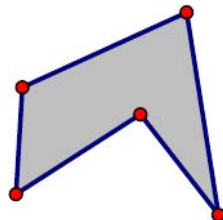
Pre-Image



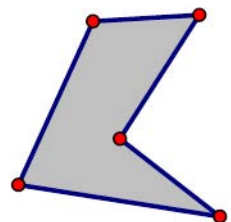
a)



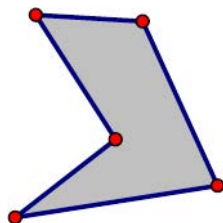
b)



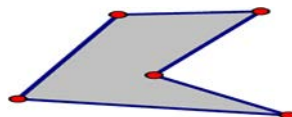
c)



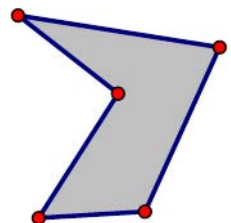
d)



e)



f)

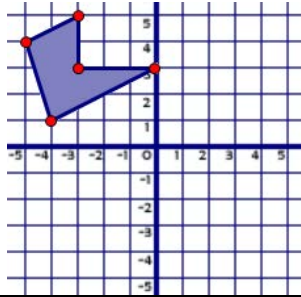
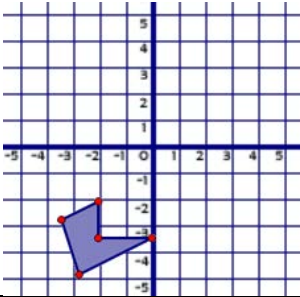
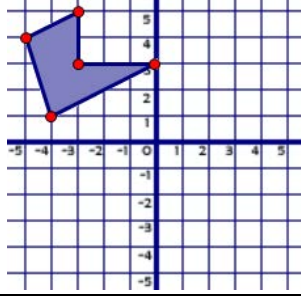
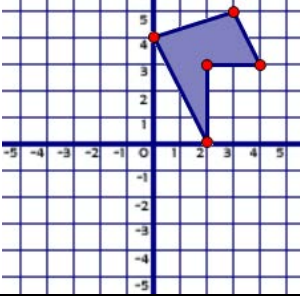
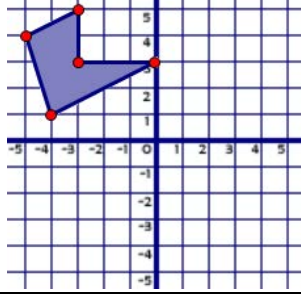
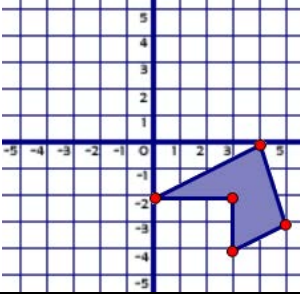
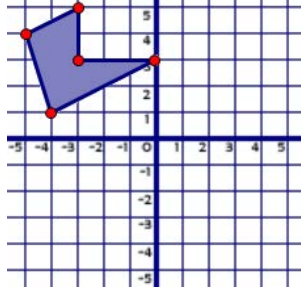
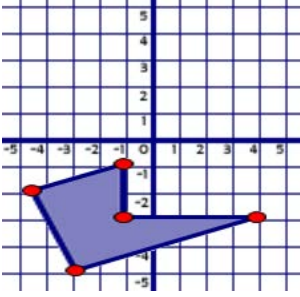
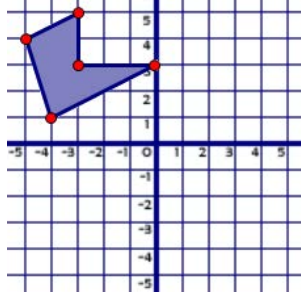
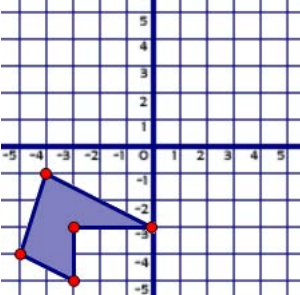


3) Which transformations are isometric?

4) Determine if the pre-image and image are isometric and also which transformation produced the image.

PRE-IMAGE	IMAGE	Circle Answer	Circle Transformation
a)		ISOMETRY NOT ISOMETRY	Rotation Reflection Translation Dilation Stretch Other
b)		ISOMETRY NOT ISOMETRY	Rotation Reflection Translation Dilation Stretch Other
c)		ISOMETRY NOT ISOMETRY	Rotation Reflection Translation Dilation Stretch Other
d)		ISOMETRY NOT ISOMETRY	Rotation Reflection Translation Dilation Stretch Other
e)		ISOMETRY NOT ISOMETRY	Rotation Reflection Translation Dilation Stretch Other

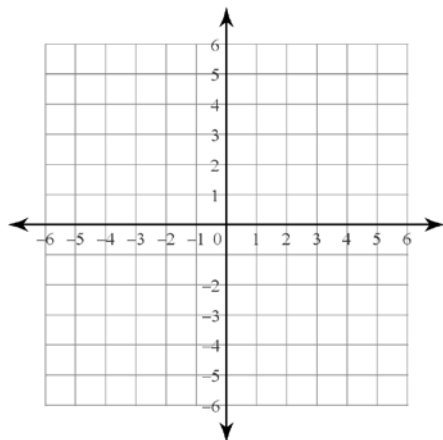
5. Determine if the pre-image and image are isometric and also which transformation produced the image.

PRE-IMAGE	IMAGE	Circle Answer	Circle Transformation
a) 		ISOMETRY NOT ISOMETRY	Rotation Reflection Translation Dilation Stretch Other
b) 		ISOMETRY NOT ISOMETRY	Rotation Reflection Translation Dilation Stretch Other
c) 		ISOMETRY NOT ISOMETRY	Rotation Reflection Translation Dilation Stretch Other
d) 		ISOMETRY NOT ISOMETRY	Rotation Reflection Translation Dilation Stretch Other
e) 		ISOMETRY NOT ISOMETRY	Rotation Reflection Translation Dilation Stretch Other

1) Translate point A to point A'.

First, graph the point $(-3, 5)$ and label it A. Now, graph the point $(2, 4)$ and label it A'.

Describe the movement from $A \rightarrow A'$.



How do the coordinates from A change when they go to A'?

Fill in the blanks: $A \rightarrow A'$

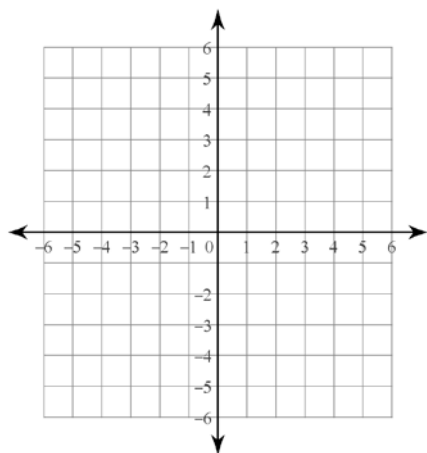
$(-3, 5) \rightarrow (-3 + \underline{\hspace{1cm}}, 5 - \underline{\hspace{1cm}})$

2) Now translate a whole triangle:

Graph and label the points $A(-3, 5)$, $B(-6, 2)$, and $C(-2, 1)$.

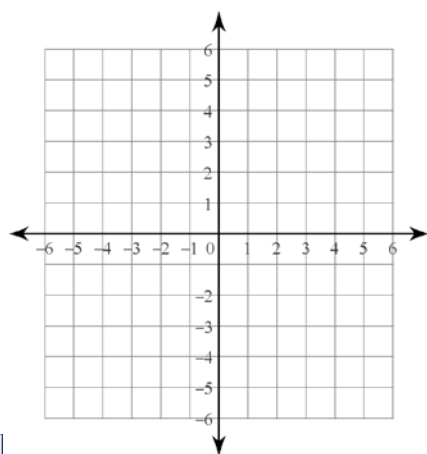
Graph and label the points $A'(2, 4)$, $B'(-1, 1)$, $C'(3, 0)$.

Describe the translation of $\triangle ABC$ to $\triangle A'B'C'$:



How do the coordinates from the points in $\triangle ABC$ change when the is translated to $\triangle A'B'C'$?

3) Translate $\triangle XYZ$, where $X(-5, 2)$, $Y(-4, 1)$, $Z(0, -3)$.



First, graph and label $\triangle XYZ$.

Next, translate $\triangle XYZ \rightarrow \triangle X'Y'Z'$ by the rule:

add 3 to the x value, and subtract 2 from they y value.

This is written as: $(x, y) \rightarrow (x + 3, y - 2)$.

Graph and label $\triangle X'Y'Z'$ by the rule: $(x, y) \rightarrow (x + 3, y - 2)$

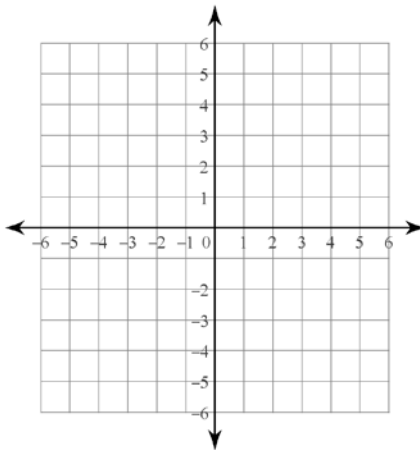
Write the points for $\triangle X'Y'Z'$:

$X'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$, $Y'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$, $Z'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$

Did the shape of the triangle change?



4) Translate $\triangle QRS$, where $Q(4, -3)$, $R(-1, 0)$, $S(2, -5)$.



Graph and label $\triangle QRS$.

Now translate $\triangle QRS \rightarrow \triangle Q'R'S'$ by the rule:
subtract 4 from the x value, and add 3 to the y value.

Graph and label $\triangle Q'R'S'$ by the rule

Fill in the blanks for this translation: $(x, y) \rightarrow (x \underline{\hspace{1cm}}, y \underline{\hspace{1cm}})$.

Write the points for $\triangle Q'R'S'$:

$Q'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$, $R'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$, $S'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$

Did the shape of the triangle change?

5) If you were asked to translate $\triangle LMN \rightarrow \triangle L'M'N'$ by the rule: $(x, y) \rightarrow (x - 5, y + 4)$, and L is at (a, b) , M is at (c, d) and N is at (e, f) , how would you write the coordinates for $\triangle L'M'N'$?

$L'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$, $M'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$, $N'(\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$

Using words, describe the translation that would be made by the rule: $(x, y) \rightarrow (x - 5, y + 4)$

**6) Using words, describe the translation that would be made by the rule: $(x, y) \rightarrow (x + 9, y - 3)$.
(How would this change the position of a triangle?)**

**7) Using words, describe the translation that would be made by the rule: $(x, y) \rightarrow (x - 1, y + 0)$.
(How would this change the position of a triangle?)**

**8) Using words, describe the translation that would be made by the rule: $(x, y) \rightarrow (x + 6, y - 2)$.
(How would this change the position of a triangle?)**

2



2

MATHEMATICS TEST

60 Minutes—60 Questions

DIRECTIONS: Solve each problem, choose the correct answer, and then fill in the corresponding oval on your answer document.

Do not linger over problems that take too much time. Solve as many as you can; then return to the others in the time you have left for this test.

You are permitted to use a calculator on this test. You may use your calculator for any problems you choose,

but some of the problems may best be done without using a calculator.

Note: Unless otherwise stated, all of the following should be assumed.

1. Illustrative figures are NOT necessarily drawn to scale.
2. Geometric figures lie in a plane.
3. The word *line* indicates a straight line.
4. The word *average* indicates arithmetic mean.

1. Marcus's favorite casserole recipe requires 3 eggs and makes 6 servings. Marcus will modify the recipe by using 5 eggs and increasing all other ingredients in the recipe proportionally. What is the total number of servings the modified recipe will make?

A. 6
B. 8
C. 10
D. 12
E. 15

2. The 35-member History Club is meeting to choose a student government representative. The members decide that the representative, who will be chosen at random, CANNOT be any of the 3 officers of the club. What is the probability that Hiroko, who is a member of the club but NOT an officer, will be chosen?

F. 0
G. $\frac{4}{35}$
H. $\frac{1}{35}$
J. $\frac{1}{3}$
K. $\frac{1}{32}$

3. For what value of x is the equation $2^{2x+7} = 2^{15}$ true?

A. 2
B. 4
C. 11
D. 16
E. 44

4. Let the function f be defined as $f(x) = 5x^2 - 7(4x + 3)$. What is the value of $f(3)$?

F. -18
G. -26
H. -33
J. -60
K. -75

5. A wallet containing 5 five-dollar bills, 7 ten-dollar bills, and 8 twenty-dollar bills is found and returned to its owner. The wallet's owner will reward the finder with 1 bill drawn randomly from the wallet. What is the probability that the bill drawn will be a twenty-dollar bill?

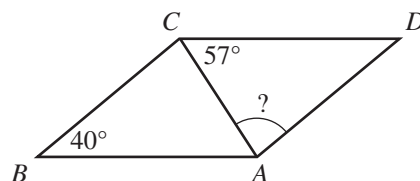
A. $\frac{1}{20}$
B. $\frac{4}{51}$
C. $\frac{1}{8}$
D. $\frac{2}{5}$
E. $\frac{2}{3}$

6. The ABC Book Club charges a \$40 monthly fee, plus \$2 per book read in that month. The Easy Book Club charges a \$35 monthly fee, plus \$3 per book read in that month. For each club, how many books must be read in 1 month for the total charges from each club to be equal?

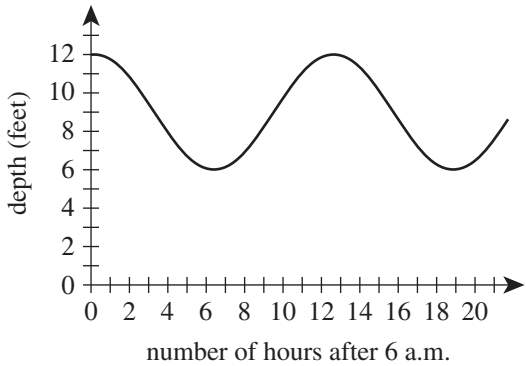
F. 1
G. 4
H. 5
J. 6
K. 75

7. In parallelogram $ABCD$ below, \overline{AC} is a diagonal, the measure of $\angle ABC$ is 40° , and the measure of $\angle ACD$ is 57° . What is the measure of $\angle CAD$?

A. 40°
B. 57°
C. 77°
D. 83°
E. 97°





8. When $x = \frac{1}{2}$, what is the value of $\frac{8x-3}{x}$?
- F. $\frac{1}{2}$
 G. 2
 H. $\frac{5}{2}$
 J. 5
 K. 10
9. In the standard (x,y) coordinate plane, what is the midpoint of the line segment that has endpoints $(3,8)$ and $(1,-4)$?
- A. $(-2,-12)$
 B. $(-1, -6)$
 C. $(\frac{11}{2}, -\frac{3}{2})$
 D. $(2, 2)$
 E. $(4,-12)$
10. The fluctuation of water depth at a pier is shown in the figure below. One of the following values gives the positive difference, in feet, between the greatest water depth and the least water depth shown in this graph. Which value is it?
- 
- F. 3
 G. 6
 H. 9
 J. 12
 K. 19
11. What is the slope of the line through $(-2,1)$ and $(2,-5)$ in the standard (x,y) coordinate plane?
- A. $\frac{3}{2}$
 B. 1
 C. -1
 D. $-\frac{3}{2}$
 E. -4
12. In Cherokee County, the fine for speeding is \$17 for each mile per hour the driver is traveling over the posted speed limit. In Cherokee County, Kirk was fined \$221 for speeding on a road with a posted speed limit of 30 mph. Kirk was fined for traveling at what speed, in miles per hour?
- F. 13
 G. 17
 H. 43
 J. 47
 K. 60
13. What is the sum of the solutions of the 2 equations below?
- $$\begin{aligned} 8x &= 12 \\ 2y + 10 &= 22 \end{aligned}$$
- A. $2\frac{2}{5}$
 B. $7\frac{1}{2}$
 C. 9
 D. 10
 E. $17\frac{1}{2}$
14. The average of 5 distinct scores has the same value as the median of the 5 scores. The sum of the 5 scores is 420. What is the sum of the 4 scores that are NOT the median?
- F. 315
 G. 320
 H. 336
 J. 350
 K. 360
15. What is the value of the expression below?
- $$| -8 + 4 | - | 3 - 9 |$$
- A. -18
 B. -2
 C. 0
 D. 2
 E. 18
16. Which of the following expressions is equivalent to $x^{\frac{2}{3}}$?
- F. $\frac{x^2}{3}$
 G. $\frac{x(2)}{3}$
 H. $\sqrt{x^3}$
 J. $\sqrt[3]{x}$
 K. $\sqrt[3]{x^2}$



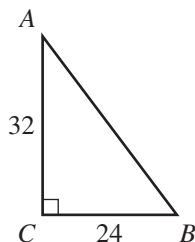
17. In the standard (x,y) coordinate plane, what is the slope of the line given by the equation $4x = 7y + 5$?

A. $-\frac{4}{7}$
 B. $\frac{4}{7}$
 C. $\frac{7}{4}$
 D. 4
 E. 7

18. For which of the following conditions will the sum of integers m and n *always* be an odd integer?

F. m is an odd integer.
 G. n is an odd integer.
 H. m and n are both odd integers.
 J. m and n are both even integers.
 K. m is an odd integer and n is an even integer.

19. The lengths of the 2 legs of right triangle $\triangle ABC$ shown below are given in inches. The midpoint of \overline{AB} is how many inches from A ?



A. 16
 B. 20
 C. 21
 D. 28
 E. 40

20. In $\triangle DEF$, the length of \overline{DE} is $\sqrt{30}$ inches, and the length of \overline{EF} is 3 inches. If it can be determined, what is the length, in inches, of \overline{DF} ?

F. 3
 G. $\sqrt{30}$
 H. $\sqrt{33}$
 J. $\sqrt{39}$
 K. Cannot be determined from the given information

21. Laura plans to paint the 8-foot-high rectangular walls of her room, and before she buys paint she needs to know the area of the wall surface to be painted. Two walls are 10 feet wide, and the other 2 walls are 15 feet wide. The combined area of the 1 window and the 1 door in her room is 60 square feet. What is the area, in square feet, of the wall surface Laura plans to paint?

A. 200
 B. 340
 C. 360
 D. 390
 E. 400

22. The length of a rectangle is 5 inches longer than the width. The perimeter of the rectangle is 40 inches. What is the width of the rectangle, in inches?

F. 7.5
 G. 8
 H. 15
 J. 16
 K. 17.5

23. 8% of 60 is $\frac{1}{5}$ of what number?

A. 0.96
 B. 12
 C. 24
 D. 240
 E. 3,750

24. Armin is trying to decide whether to buy a season pass to his college basketball team's 20 home games this season. The cost of an individual ticket is \$14, and the cost of a season pass is \$175. The season pass will admit Armin to any home basketball game at no additional cost. What is the minimum number of home basketball games Armin must attend this season in order for the cost of a season pass to be less than the total cost of buying an individual ticket for each game he attends?

F. 8
 G. 9
 H. 12
 J. 13
 K. 20

25. $\frac{4.8 \times 10^{-7}}{1.6 \times 10^{-11}} = ?$

A. 3.0×10^4
 B. 3.0×10^{-4}
 C. 3.0×10^{-18}
 D. 3.2×10^{18}
 E. 3.2×10^4

26. A circle in the standard (x,y) coordinate plane has center $C(-1,2)$ and passes through $A(2,6)$. Line segment \overline{AB} is a diameter of this circle. What are the coordinates of point B ?

F. $(-6,-2)$
 G. $(-5,-1)$
 H. $(-4,-2)$
 J. $(4, 2)$
 K. $(5,10)$

27. Which of the following expressions is a factor of $x^3 - 64$?

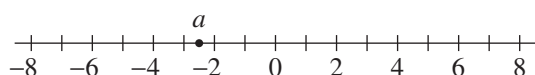
A. $x - 4$
 B. $x + 4$
 C. $x + 64$
 D. $x^2 + 16$
 E. $x^2 - 4x + 16$



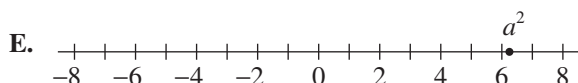
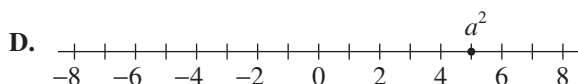
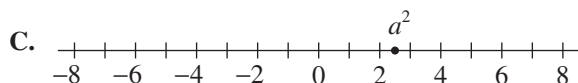
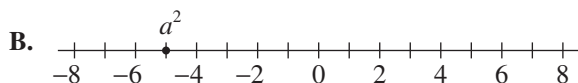
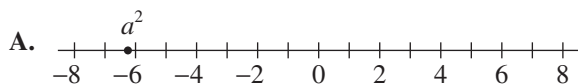
28. The average of a list of 4 numbers is 90.0. A new list of 4 numbers has the same first 3 numbers as the original list, but the fourth number in the original list is 80, and the fourth number in the new list is 96. What is the average of this new list of numbers?

F. 90.0
G. 91.5
H. 94.0
J. 94.5
K. 94.8

29. The number a is located at -2.5 on the number line below.



One of the following number lines shows the location of a^2 . Which number line is it?



30. Maria ordered a pizza. She ate only $\frac{2}{9}$ of it and gave the remaining pizza to her 3 brothers. What fraction of the whole pizza will each of Maria's brothers receive, if they share the remaining pizza equally?

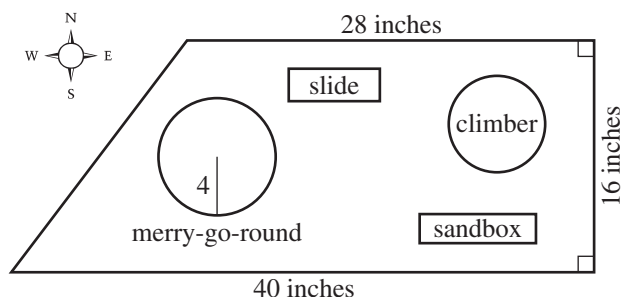
F. $\frac{7}{9}$
G. $\frac{3}{7}$
H. $\frac{1}{3}$
J. $\frac{7}{27}$
K. $\frac{2}{27}$

31. The number 1,001 is the product of the prime numbers 7, 11, and 13. Knowing this, what is the prime factorization of 30,030?

A. $3 \cdot 7 \cdot 10 \cdot 13$
B. $30 \cdot 7 \cdot 11 \cdot 13$
C. $2 \cdot 5 \cdot 7 \cdot 11 \cdot 13$
D. $3 \cdot 7 \cdot 10 \cdot 11 \cdot 13$
E. $2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$

Use the following information to answer questions 32–34.

Mikea, an intern with the Parks and Recreation Department, is developing a proposal for the new trapezoidal Springdale Park. The figure below shows her scale drawing of the proposed park with 3 side lengths and the radius of the merry-go-round given in inches. In Mikea's scale drawing, 1 inch represents 1.5 feet.



32. What is the area, in square inches, of the scale drawing of the park?

F. 448
G. 544
H. 640
J. 672
K. 1,088

33. Mikea's proposal includes installing a fence on the perimeter of the park. What is the perimeter, in *feet*, of the park?

A. 84
B. 88
C. 104
D. 126
E. 156

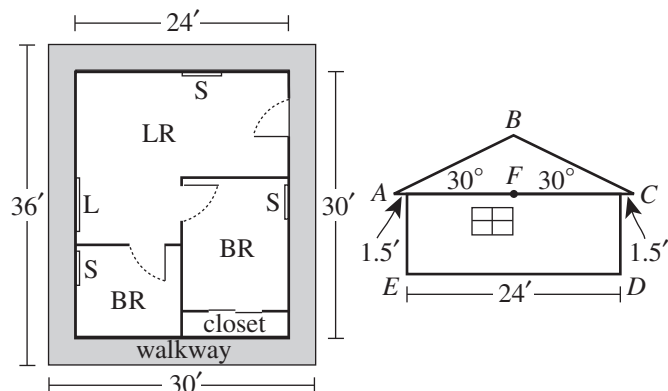
34. The length of the south side of the park is what percent of the length of the north side?

F. 112%
G. 124%
H. $142\frac{6}{7}\%$
J. 175%
K. 250%



Use the following information to answer questions 35–37.

The Smith family is planning to build a 3-room cabin which consists of 2 bedrooms (BR) and 1 living room (LR). Shown below are the rectangular floor plan (left figure) and a side view of the cabin (right figure). In the side view, the roof forms an isosceles triangle ($\triangle ABC$), the walls are perpendicular to the level floor (\overline{ED}), $\overline{AC} \parallel \overline{ED}$, F is the midpoint of \overline{AC} , and $\overline{BF} \perp \overline{AC}$.



During the week the Smiths plan to roof the cabin, there is a 20% chance of rain each day.

35. Mr. Smith plans to build a 3-foot-wide walkway around the outside of the cabin, as shown in the floor plan. What will be the area, in square feet, of the top surface of the walkway?

A. 171
B. 324
C. 360
D. 396
E. 720

36. Mrs. Smith will install a ceiling fan in each room of the cabin and will place curtains over the 4 windows. Each of the ceiling fans has a price of \$52.00. The price of curtains for each small window (S) is \$39.50, and the price of curtains for the large window (L) is twice that for the small window. Based on this information, which of the following values is closest to the total price Mrs. Smith will pay for curtains and ceiling fans?

F. \$262
G. \$302
H. \$341
J. \$354
K. \$393

37. Mr. and Mrs. Smith plan to roof the cabin on 2 consecutive days. Assuming that the chance of rain is independent of the day, what is the probability that it will rain both days?

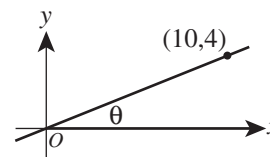
A. 0.04
B. 0.08
C. 0.16
D. 0.20
E. 0.40

38. Which of the following expressions, when evaluated, equals an irrational number?

F. $\frac{\sqrt{2}}{\sqrt{8}}$
G. $\frac{\sqrt{8}}{\sqrt{2}}$
H. $(\sqrt{8})^2$
J. $\sqrt{2} \times \sqrt{8}$
K. $\sqrt{2} + \sqrt{8}$

39. A line through the origin and (10,4) is shown in the standard (x,y) coordinate plane below. The acute angle between the line and the positive x -axis has measure θ . What is the value of $\tan \theta$?

A. $\frac{\sqrt{29}}{2}$
B. $\frac{2}{\sqrt{29}}$
C. $\frac{5}{\sqrt{29}}$
D. $\frac{2}{5}$
E. $\frac{5}{2}$



40. The equation $|2x - 8| + 3 = 5$ has 2 solutions. Those solutions are equal to the solutions to which of the following pairs of equations?

F. $2x - 5 = 5$
 $-2x - 5 = -5$
G. $2x - 8 = 2$
 $-2x - 8 = 2$
H. $2x - 8 = 8$
 $-(2x - 8) = 8$
J. $2x - 8 = 2$
 $-(2x - 8) = 8$
K. $2x - 8 = 2$
 $-(2x - 8) = 2$



41. The frequency chart below shows the cumulative number of Ms. Hernandez's science students whose test scores fell within certain score ranges. All test scores are whole numbers.

Score range	Cumulative number of students
65–70	12
65–80	13
65–90	19
65–100	21

How many students have a test score in the interval 71–80 ?

- A. 1
B. 6
C. 8
D. 12
E. 13
42. The number of decibels, d , produced by an audio source can be modeled by the equation $d = 10 \log\left(\frac{I}{K}\right)$, where I is the sound intensity of the audio source and K is a constant. How many decibels are produced by an audio source whose sound intensity is 1,000 times the value of K ?
- F. 4
G. 30
H. 40
J. 100
K. 10,000
43. Mario plays basketball on a town league team. The table below gives Mario's scoring statistics for last season. How many points did Mario score playing basketball last season?

Type of shot	Number attempted	Percent successful
1-point free throw	80	75%
2-point field goal	60	90%
3-point field goal	60	25%

- A. 129
B. 190
C. 213
D. 330
E. 380

44. The graph of $y = |x - 6|$ is in the standard (x, y) coordinate plane. Which of the following transformations, when applied to the graph of $y = |x|$, results in the graph of $y = |x - 6|$?

- F. Translation to the right 6 coordinate units
G. Translation to the left 6 coordinate units
H. Translation up 6 coordinate units
J. Translation down 6 coordinate units
K. Reflection across the line $x = 6$

45. Toby wants to find the volume of a solid toy soldier. He fills a rectangular container 8 cm long, 6 cm wide, and 10 cm high with water to a depth of 4 cm. Toby totally submerges the toy soldier in the water. The height of the water with the submerged toy soldier is 6.6 cm. Which of the following is closest to the volume, in cubic centimeters, of the toy soldier?

- A. 125
B. 156
C. 192
D. 208
E. 317

46. A box in the shape of a cube has an interior side length of 18 inches and is used to ship a right circular cylinder with a radius of 6 inches and a height of 12 inches. The interior of the box not occupied by the cylinder is filled with packing material. Which of the following numerical expressions gives the number of cubic inches of the box filled with packing material?

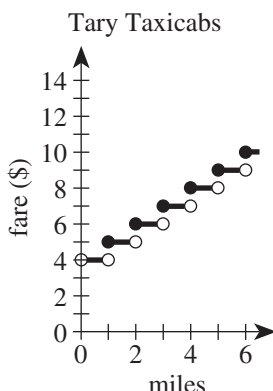
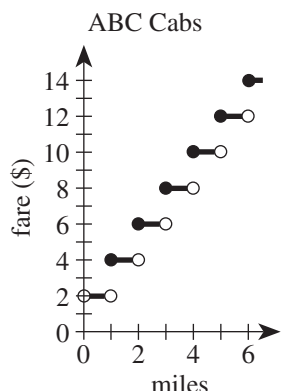
- F. $6(18)^2 - 2\pi(6)(12) - 2\pi(6)^2$
G. $6(18)^2 - 2\pi(6)(12)$
H. $18^3 - \pi(6)(12)^2$
J. $18^3 - \pi(6)^2(12)$
K. $18^3 - \pi(12)^3$

47. A room has a rectangular floor that is 15 feet by 21 feet. What is the area of the floor in square yards ?

- A. 24
B. 35
C. 36
D. 105
E. 144



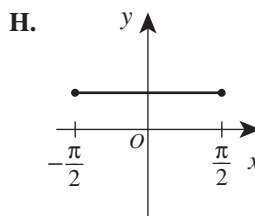
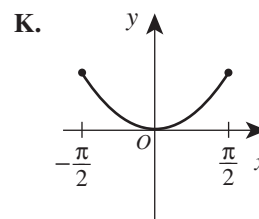
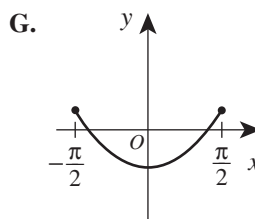
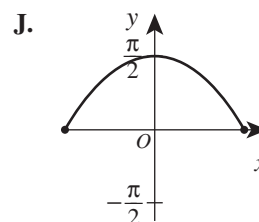
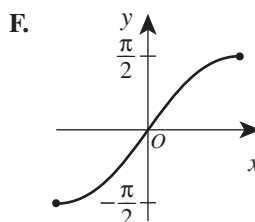
48. ABC Cabs and Tary Taxicabs both have an initial fare of a whole number of dollars for 1 passenger. The fare increases a whole number of dollars at each whole number of miles traveled. The graphs below show the 1-passenger fares, in dollars, for both cab companies for trips up to 6 miles. When the fares of the 2 cab companies are compared, what is the cheaper fare for a 5-mile trip?



- F. \$ 8
G. \$ 9
H. \$10
J. \$11
K. \$12
49. The graph of a function $y = f(x)$ consists of 3 line segments. The graph and the coordinates of the endpoints of the 3 line segments are shown in the standard (x,y) coordinate plane below. What is the area, in square coordinate units, of the region bounded by the graph of $y = f(x)$, the positive y -axis, and the positive x -axis?
-
50. The sum of 2 positive numbers is 151. The lesser number is 19 more than the square root of the greater number. What is the value of the greater number minus the lesser number?
51. The list of numbers 41, 35, 30, X , Y , 15 has a median of 25. The mode of the list of numbers is 15. To the nearest whole number, what is the mean of the list?
52. You are given the following system of equations:
- $$y = x^2$$
- $$rx + sy = t$$
- where r , s , and t are integers. For which of the following will there be more than one (x,y) solution, with real-number coordinates, for the system?
53. The 3rd and 4th terms of an arithmetic sequence are 13 and 18, respectively. What is the 50th term of the sequence?
54. One of the following graphs in the standard (x,y) coordinate plane is the graph of $y = \sin^2 x + \cos^2 x$ over the domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$. Which one?

- A. 10
B. 13
C. 14
D. 15
E. 20

- F. 19
G. 66
H. 85
J. 91
K. 121





55. What is the period of the function $f(x) = \csc(4x)$?

- A. π
- B. 2π
- C. 4π
- D. $\frac{\pi}{4}$
- E. $\frac{\pi}{2}$

56. At the school carnival, Mike will play a game in which he will toss a penny, a nickel, and a dime at the same time. He will be awarded 3 points for each coin that lands with heads faceup. Let the random variable x represent the total number of points awarded on any toss of the coins. What is the expected value of x ?

- F. 1
- G. $\frac{3}{2}$
- H. $\frac{9}{2}$
- J. 6
- K. 9

57. For what positive real value of k , if any, is the determinant of the matrix $\begin{bmatrix} k & 4 \\ 3 & k \end{bmatrix}$ equal to k ?

(Note: The determinant of matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ equals $ad - bc$.)

- A. 3
- B. 4
- C. 12
- D. $\sqrt{12}$
- E. There is no such value of k .

58. Given a positive integer n such that $i^n = 1$, which of the following statements about n must be true?

(Note: $i^2 = -1$)

- F. When n is divided by 4, the remainder is 0.
- G. When n is divided by 4, the remainder is 1.
- H. When n is divided by 4, the remainder is 2.
- J. When n is divided by 4, the remainder is 3.
- K. Cannot be determined from the given information

59. For $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$, $|\sin \theta| \geq 1$ is true for all and only the values of θ in which of the following sets?

- A. $\left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$
- B. $\left\{\frac{\pi}{2}\right\}$
- C. $\left\{\theta \mid -\frac{\pi}{2} < \theta < \frac{\pi}{2}\right\}$
- D. $\left\{\theta \mid -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}\right\}$
- E. The empty set

60. Ray \overrightarrow{PK} bisects $\angle LPM$, the measure of $\angle LPM$ is $11x^\circ$, and the measure of $\angle LPK$ is $(4x + 18)^\circ$. What is the measure of $\angle KPM$?

- F. 12°
- G. $28\frac{2}{7}^\circ$
- H. 42°
- J. $61\frac{1}{5}^\circ$
- K. 66°

END OF TEST 2

STOP! DO NOT TURN THE PAGE UNTIL TOLD TO DO SO.

DO NOT RETURN TO THE PREVIOUS TEST.

Key	Reporting Category*						
	PHM					IES	MDL
	N	A	F	G	S		
1. C						—	
2. K					—		—
3. B		—					
4. J			—				
5. D						—	—
6. H		—					—
7. D				—			—
8. G				—		—	
9. D				—			
10. G						—	—
11. D			—				
12. H	—						—
13. B						—	
14. H					—		—
15. D						—	
16. K	—						
17. B		—					
18. K						—	—
19. B				—			
20. K				—			—
21. B				—		—	—
22. F						—	—
23. C						—	
24. J						—	—
25. A		—					
26. H				—			—
27. A		—					
28. H						—	—
29. E						—	
30. J						—	—

Key	Reporting Category*						
	PHM					IES	MDL
	N	A	F	G	S		
31. E						—	
32. G						—	—
33. E						—	
34. H						—	
35. C						—	—
36. J						—	
37. A					—		
38. K	—						
39. D				—			
40. K		—					
41. A					—		—
42. G			—				
43. C						—	—
44. F			—				—
45. A						—	—
46. J				—			—
47. B						—	
48. G						—	—
49. B				—			
50. J		—					—
51. C					—		
52. F		—					
53. A			—				—
54. H			—				—
55. E			—				
56. H					—		—
57. B	—						
58. F	—						
59. A			—				
60. K						—	—

Combine the totals of these columns and put in the blank for PHM in the box below.

Reporting Categories*PHM** = Preparing for Higher Math

N = Number & Quantity

A = Algebra

F = Functions

G = Geometry

S = Statistics & Probability

IES = Integrating Essential Skills**MDL** = Modeling**Number Correct (Raw Score) for:**

Preparing for Higher Math (PHM)

(N + A + F + G + S)

(35)

Integrating Essential Skills (IES)

(25)

Total Number Correct for Mathematics Test

(PHM + IES)

(60)

Modeling (MDL)

(Not included in total number correct for mathematics test raw score)

(28)